

Steady-State Cosmology in the Yilmaz Theory of Gravitation

H. E. Puthoff
Institute for Advanced Studies at Austin
4030 W. Braker Ln., Suite 300
Austin, Texas 78759

Abstract

Yilmaz has proposed a modification to Einstein's equations in which the gravitational stress-energy pseudotensor is replaced by a true tensor [1,2]. Application of the Yilmaz approach to such solar system phenomena as redshift, light bending, perihelion advance and radar delay yields results indistinguishable from standard GR. Application of the Yilmaz approach to cosmological issues, however, is relatively unexplored territory. Here we show, following up on recent analysis by Ibison [3], that application of the *Cosmological Principle* (space should look the same to all co-moving observers) leads to a similar constraint with regard to the temporal coordinate (i.e., the *Perfect Cosmological Principle* – that space should look the same to all co-moving observers at all times). As a result, the Yilmaz approach is more in alignment with continuous-matter-creation, steady-state-universe models, than with singular-matter-creation, Big Bang models. Though arguments against the viability of steady-state models are well known, e.g., the need for alternative explanations of the CMB (Cosmic Microwave Background), origin of the light nuclei, and the apparent requirement for non-baryonic dark matter, progress continues to be made in addressing these key issues, as in the recent papers by Hoyle, Burbidge and Narlikar (hereafter, HBN) [4-6]. One noteworthy consequence of the Yilmaz steady-state universe examined here is that the universe is predicted to expand exponentially without introduction of an undefined “dark energy” component.

1. Introduction

In his studies of Einstein's development of general relativity (GR), Yilmaz noted that Einstein used an approximate formula in his derivation of the gravitational redshift from the (weak) principle of equivalence plus the kinematical Doppler shift - approximate in that Einstein used the Doppler formula only to first order [7]. Yilmaz investigated the consequences of replacing the first order Doppler shift expression with the exact special relativistic Doppler formula, the outcome of which was that he proposed a modification to Einstein's equations that involved introduction of an additional (true) gravitational stress-energy tensor \mathbf{t}_μ^v source term on the RHS of the Einstein equation. Thus the Yilmaz-modified Einstein equation takes the form

$$\mathbf{R}_\mu^v - \frac{1}{2} \delta_\mu^v \mathbf{R} = -\frac{8\pi G}{c^4} \mathbf{T}_\mu^v \quad (1)$$

where now $\mathbf{T}_\mu^v = \tau_\mu^v + \mathbf{t}_\mu^v$, τ_μ^v representing the usual stress-energy source terms, and \mathbf{t}_μ^v the newly introduced gravitational stress-energy term.

2. Yilmaz Procedure

In his new formulation, investigated by use of powerful symbolic manipulation programs (e.g., MACSYMA and MATHEMATICA), Yilmaz found it convenient to sidestep solving Einstein's equation directly by introducing an intermediate tensor potential φ_μ^ν (a curved-spacetime generalization of the gravitational potential) in terms of which the metric tensor could be determined. The equation given for φ_μ^ν is

$$\square^2 \varphi_\mu^\nu = \frac{4\pi G}{c^4} \tau_\mu^\nu \quad (2)$$

where \square^2 is the Beltrami operator, $\square^2 = (1/\sqrt{-g}) \partial_\alpha (\sqrt{-g} \partial^\alpha)$.¹ For a given problem φ_μ^ν is solved in terms of τ_μ^ν , and the metric $g_{\mu\nu}$ obtained from

$$g_{\mu\nu} = \left(\tilde{\eta} e^{2(\varphi_1 - 2\tilde{\varphi})} \right)_{\mu\nu} \quad (3)$$

where $\tilde{\eta}$ is the Minkowski metric, $\mathbf{1}$ is the unit tensor, $\tilde{\varphi} = \varphi_\alpha^\alpha$ is a symmetric tensor, and φ its trace.² When the Cosmological Principle holds, $\tilde{\varphi}$ is diagonal and isotropic.

3. Cosmological Metrics

To apply the Yilmaz procedure to cosmology wherein the Cosmological Principle is assumed (space looks the same to all co-moving observers), we take τ_μ^ν and φ_μ^ν to be of the form

$$\tau_\mu^\nu = \begin{pmatrix} \rho_{tot} c^2 & \\ & -p_{tot} \end{pmatrix}, \quad \varphi_\mu^\nu = \begin{pmatrix} \varphi_0^0 & 0 \\ 0 & \varphi_1^1 \end{pmatrix} \quad (4a,b)$$

where ρ_{tot} and p_{tot} are taken to be the *total* coordinate density and pressure in the co-moving Hubble flow (x and y components suppressed). We emphasize the adjective *total* because in the most general case we include contributions from matter, radiation, the vacuum, and – of significance later – a postulated matter creation field.. That is, e.g.,

¹ With Yilmaz's stated preference for the use of harmonic coordinates satisfying $\partial_\nu (\sqrt{-g} g^{\mu\nu}) = 0$, the Beltrami operator reduces to $\square^2 = \partial^\alpha \partial_\alpha$.

² The newly-defined gravitational stress-energy tensor can then be calculated from

$$\mathbf{t}_\mu^\nu = \frac{c^4}{4\pi G} \left[-2\partial_\mu \Phi_\beta^\alpha \partial^\nu \Phi_\alpha^\beta + \partial_\mu \phi \partial^\nu \phi + \delta_\mu^\nu \left(\partial^\lambda \Phi_\beta^\alpha \partial_\lambda \Phi_\alpha^\beta - \frac{1}{2} \partial^\lambda \phi \partial_\lambda \phi \right) \right]$$

$$\rho_{tot} = \rho_m + \rho_{rad} + \rho_{vac} + \rho_c \quad (5)$$

By (3) and (4b) we obtain

$$g_{\mu\nu} = \begin{pmatrix} \exp[-2(\varphi_0^0 - 3\varphi_1^1)] & 0 \\ 0 & -\exp[2(\varphi_0^0 + \varphi_1^1)] \end{pmatrix}, \quad \sqrt{-g} = \exp(2\varphi) \quad (6)$$

and corresponding line element given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \exp[-2(\varphi_0^0 - 3\varphi_1^1)] c^2 d\tau^2 - \exp[2(\varphi_0^0 + \varphi_1^1)] d\mathbf{r}^2 \quad (7)$$

As outlined in Peacock [8], metrics satisfying the Cosmological Principle can be written, up to a general coordinate transformation, in the form of the Robertson-Walker (RW) line element as

$$ds^2 = c^2 dt^2 - a^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 d\psi^2 \right) \quad (8)$$

where $a(t)$ is a scale factor and k is the curvature of space. Given that, cosmologically, space is observed to be flat ($k = 0$), this can be rewritten as

$$ds^2 = c^2 dt^2 - a^2(t) d\mathbf{r}^2 \quad (9)$$

Application of Yilmaz's preferred choice of harmonic coordinates satisfying $\partial_\nu(\sqrt{-g} g^{\mu\nu}) = 0$, and allowing for no spatial variations (as required by the Cosmological Principle applied to a flat space), leads to $\varphi_0^0 = 0$, in which case (7) reduces to (with $d\tau_h$ now a harmonic coordinate time interval)

$$ds^2 = \exp(6\varphi_1^1) c^2 d\tau_h^2 - \exp(2\varphi_1^1) d\mathbf{r}^2 \quad (10)$$

Comparison of (10) and (9) reveals that the respective time coordinates are simply related by the coordinate transformation

$$dt = a^3 d\tau_h \quad (11)$$

with the scale factor a related to φ_1^1 by

$$\varphi_1^1 = \ln a \quad (12)$$

in which case (10) can also be written in terms of the harmonic time coordinate as

$$ds^2 = a^6 c^2 d\tau_h^2 - a^2 d\mathbf{r}^2 \quad (13)$$

4. Cosmological Solutions

With the above in place, we are now in a position to substitute (4) into (2), taking into account the form of the Beltrami operator in harmonic coordinates (footnote 1), the fact that use of harmonic coordinates (allowing for no spatial variations) led to $\varphi_0^0 = 0$, and by (12) $\varphi_1^1 = \ln a$. The result is

$$\frac{1}{a^6} \frac{d^2}{c^2 d\tau_h^2} \begin{pmatrix} 0 & 0 \\ 0 & \ln a \end{pmatrix} = \frac{4\pi G}{c^4} \begin{pmatrix} \rho_{tot} c^2 & 0 \\ 0 & -p_{tot} \end{pmatrix} \quad (14)$$

yielding for the first and second Yilmaz-Friedmann equations (see also Appendix A)

$$\rho_{tot} = 0, \quad \frac{d^2}{d\tau_h^2} (\ln a) = -\frac{4\pi G}{c^2} a^6 p_{tot} \quad (15a,b)$$

Use of the coordinate transformation (11) allows (15b) to be rewritten in the form

$$a \frac{d^2 a}{dt^2} + 2 \left(\frac{da}{dt} \right)^2 = -\frac{4\pi G}{c^2} a^2 p_{tot} \quad (16)$$

with solution

$$a = e^{H_0 t} \quad (17)$$

where

$$p_{tot} = -\frac{3H_0^2 c^2}{4\pi G} \quad (18)$$

With these equations in place, we are now in a position to evaluate their significance and implications.

5. Steady-State Yilmaz Cosmology

We now show that the Yilmaz-Friedmann equations (15a,b) and resulting solution (17) indicate that, for a finite mass density universe ($\rho_m \neq 0$), the Cosmological Principle applied to the Yilmaz theory leads unambiguously to a non-matter-conserving, steady-state solution that satisfies the Perfect Cosmological Principle, i.e., that coordinate density (in this case, $\rho_{tot} = 0$) and pressure remain constant as the universe expands (exponentially). Neglecting to first order (a) the pressures and energy densities associated with radiation and the cosmological constant (negligible compared to that of matter in a steady-state universe), and (b) the pressure associated with matter, the above Friedmann equation requirements generated by application of the Cosmological Principle to the Yilmaz theory can be met by postulating the (average) continuous creation of

matter by a vacuum field with negative energy density and negative pressure such that (15a) and (18) are satisfied. Just such a scalar vacuum field (C -field, for creation field – see, e.g., [8], p. 81) has been explored in some detail in a series of papers by HBN [4-6] to delineate classical steady-state and quasi-steady-state cosmologies. In such modeling the scalar C -field exerts a negative pressure (as in (18)) that drives universe expansion in a manner not unlike that assumed in inflationary models. The field also acts as a negative energy density source in the creation process, balancing the positive energy of matter production during expansion so as to maintain constant matter energy density (i.e., $\rho_{tot}c^2 = (\rho_m + \rho_C)c^2 = 0$). For the steady-state Yilmaz cosmology case at hand, modeling along HBN lines yields (for $\rho_C c^2 = p_C < 0$) [4]

$$p_{tot} = p_C = -\dot{C}^2 = -\frac{3H_0^2 c^2}{4\pi G}, \quad \rho_C c^2 = -\dot{C}^2 \Rightarrow \rho_m c^2 = -\rho_C c^2 = \frac{3H_0^2 c^2}{4\pi G} \quad (19)$$

where the matter creation rate represented by \dot{C} maintains constant (negative) pressure and (positive) matter density. Though steady-state cosmologies in general, and the HBN approach in particular, remain controversial, nascent models of the creation process are under development both theoretically in terms of proposed action principles, and with regard to mechanisms based on quantum vacuum principles applicable in a curved space.

6. Conclusions

Application of the Cosmological Principle (space should look the same to all co-moving observers) to the Yilmaz-modified form of GR leads unambiguously to a universe in which total energy density vanishes. Of the various cosmology models under consideration in the literature, only a particular steady-state model meets this criterion for finite matter density. It is a universe in which the Perfect Cosmological Principle is satisfied (that space should look the same to all co-moving observers at all times), and whose expansion is exponential. To maintain constant coordinate mass density, such a universe must needs embody an average continuous low-level, negative-energy, matter-creation process in lieu of a singular Big Bang process, models for which are being explored in the literature. Since details on such a process have yet to be elaborated in detail, and other factors supporting such a model (e.g., source of the CMB, origin of the light nuclei, etc.) remain ambiguous, it cannot be said that the Yilmaz-modified form of GR has been shown to fit observational data, only that for the cosmological model explored here it cannot be ruled out.

Acknowledgements

The author is grateful to Michael Ibison, Eric Davis, Huseyin Yilmaz and Carroll Alley for useful discussions during this study.

References

- [1] H. Yilmaz, "Relativity and quantum mechanics," *Int. Jour. Theor. Phys.* **21**, 871-902 (1982)
- [2] H. Yilmaz, "Toward a field theory of gravitation," *Il Nuovo Cimento* **107 B**, 941-960 (1992).
- [3] M. Ibison, "Cosmological test of the Yilmaz theory of gravity," preprint (2005).
- [4] F. Hoyle, G. Burbidge, and J. V. Narlikar, "Further astrophysical quantities expected in a quasi-steady state Universe," *Astron. Astrophys.* **289**, 729-739 (1994).
- [5] F. Hoyle, G. Burbidge, and J. V. Narlikar, "The basic theory underlying the quasi-steady-state cosmology," *Proc. Math. and Phys. Sciences* **448**, 191-212 (1995).
- [6] G. Burbidge, F. Hoyle, and J. V. Narlikar, "A different approach to cosmology," *Phys. Today*, 38-44 (April 1999).
- [7] H. Yilmaz, "New Theory of Gravitation," in *Proc. of the 4th Marcel Grossmann Meeting on General Relativity*, R. Ruffini, ed. (Elsevier Science Publ. B.V., 1986), pp. 1753-1791.
- [8] J. A. Peacock, *Cosmological Physics* (Cambridge University Press, Cambridge, 1999), p. 70.

In GR cosmology studies, the first Friedmann equation is given in the canonical form (for flat space, $k = 0$) as

$$\frac{1}{c^2} \left(\frac{da}{dt} \right)^2 - \left(\frac{8\pi G}{3c^4} \right) \rho_{tot} c^2 a^2 = 0 \quad (\text{A-1})$$

In Yilmaz-modified GR, we find that the first Yilmaz-Friedmann equation takes the form (15a),

$$\rho_{tot} = 0 \quad (\text{A-2})$$

To express this in canonical form, we note that substitution of (12) into the definition of \mathbf{t}_μ^{ν} given in Footnote 1 yields the identity

$$t_0^0 \equiv \left(\frac{3c^4}{8\pi G} \right) \frac{1}{c^2} \frac{1}{a^2} \left(\frac{da}{dt} \right)^2 \quad (\text{A-3})$$

The first Yilmaz-Friedmann equation can then be formally rewritten in canonical form as

$$\frac{1}{c^2} \left(\frac{da}{dt} \right)^2 - \left(\frac{8\pi G}{3c^4} \right) (\rho_{tot} c^2 + t_0^0) a^2 = 0 \quad (\text{A-4})$$

The significance of the $t_0^0 \propto (\dot{a}/a)^2$ identity is that the Yilmaz-added gravitational stress-energy term serves to cancel the energy contribution associated with expansion in standard GR.

Similarly, in standard GR, the second Friedmann equation is given in canonical form as

$$\frac{1}{c^2} \frac{1}{a} \frac{d^2 a}{dt^2} = - \left(\frac{4\pi G}{3c^4} \right) (\rho_{tot} c^2 + 3p_{tot}) \quad (\text{A-5})$$

In contrast, in Yilmaz-modified GR, the second Yilmaz-Friedmann equation takes the form (16),

$$\frac{1}{c^2} \frac{1}{a} \frac{d^2 a}{dt^2} + \frac{1}{c^2} \frac{2}{a^2} \left(\frac{da}{dt} \right)^2 = - \frac{4\pi G}{c^4} p_{tot} \quad (\text{A-6})$$

Again, taking into account that in Yilmaz-modified GR the added gravitational stress energy t_0^0 has an associated pressure (Footnote 1) $-t_1^1 = t_0^0$, the second Yilmaz-Friedmann equation can be formally expressed in canonical form as

$$\frac{1}{c^2} \frac{1}{a} \frac{d^2 a}{dt^2} = - \left(\frac{4\pi G}{3c^4} \right) \left[(\rho_{tot} c^2 + t_0^0) + 3(p_{tot} - t_1^1) \right] \quad (\text{A-7})$$

These rewritten forms of the Yilmaz-Friedmann equations serve to highlight the significance of the Yilmaz addition of \mathbf{t}_μ^ν ; namely, cancellation of the $(\dot{a}/a)^2$ energy contribution in the first (standard) Friedmann equation, but generation of additive contributions to pressure $\propto (\dot{a}/a)^2$ in the second.