POLARIZABLE-VACUUM APPROACH TO GENERAL RELATIVITY

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1. Abstract

Topics in general relativity (GR) are routinely treated in terms of tensor formulations in curved spacetime. An alternative approach is presented here, based on treating the vacuum as a polarizable medium. Beyond simply reproducing the standard weak-field predictions of GR, the polarizable vacuum (PV) approach provides additional insight into what is meant by a curved metric. For the strong field case, a divergence of predictions in the two formalisms (GR vs. PV) provides fertile ground for both laboratory and astrophysical tests.

2. Introduction

The principles of General Relativity (GR) are generally formulated in terms of tensor formulations in curved spacetime. Such an approach captures in a concise and elegant way the interaction between masses, and their consequent motion. "Matter tells space how to curve, and space tells matter how to move [1]." During the course of development of GR over the years, however, alternative approaches have emerged that provide convenient methodologies for investigating metric changes in other formalisms, and which yield heuristic insight into what is meant by a curved metric.

One approach that has intuitive appeal is the polarizable-vacuum (PV) approach [2-3]. The PV approach treats metric changes in terms of equivalent changes in the permittivity and permeability constants of the vacuum, e_o and μ_o, essentially along the lines of the so-called *"THeμ"* methodology used in comparative studies of gravitational theories [4-6].

In brief, Maxwell's equations in curved space are treated in the isomorphism of a polarizable medium of variable refractive index in flat space [7]; the bending of a light ray near a massive body is modeled as due to an induced spatial variation in the refractive index of the vacuum near the body; the reduction in the velocity of light in a gravitational potential is represented by an effective increase in the refractive index of the vacuum, and so forth. As elaborated in Refs. [3-7], PV modeling can be carried out in a self-consistent way so as to reproduce to appropriate order both the equations of GR, and the match to the classical experimental tests of those equations. Under conditions of extreme metric perturbation, however, the PV approach predicts certain results at variance with the standard GR approach. We discuss these variances in terms of testable implications, both in the laboratory and with regard to astrophysical consequences.

3. The Polarizable Vacuum

The electric flux vector ***D*** in a linear, homogeneous medium can be written

$$\mathbf{D} = e\mathbf{E} = e_o\mathbf{E} + \mathbf{P} = e_o\mathbf{E} + a_V\mathbf{E} \,, \tag{1}$$

where e and e_O are the permittivities of the medium and of the vacuum, respectively, and the polarization ***P*** corresponds to the induced dipole moment per unit volume in the medium whose polarizability per unit volume is a_V. The identical form of the last two terms leads naturally to the interpretation of e_O as the polarizability per unit volume of the vacuum, treated as a medium in its own right. This interpretation is explicitly corroborated in detail by the quantum picture of the vacuum where it is shown that the vacuum acts as a polarizable medium by virtue of induced dipole moments resulting from the excitation of virtual electron-positron pairs [8].

To represent curved-space conditions, the basic postulate of the PV approach is that the polarizability of the vacuum in the vicinity of a mass (or other mass-energy concentrations) differs from its asymptotic far-field value by virtue of vacuum polarization effects induced by the presence of the mass. That is, we postulate for the vacuum itself

$$\mathbf{D} = e\mathbf{E} = Ke_o\mathbf{E} \,, \tag{2}$$

where K is the (altered) dielectric constant of the vacuum (typically a function of position) due to (GR-induced) vacuum polarizability changes under consideration. *Throughout the rest of our study the vacuum dielectric constant K constitutes the key variable of interest.*

3.1. VELOCITY OF LIGHT IN A VACUUM OF VARIABLE POLARIZABILITY

In this section we examine quantitatively the effects of a polarizable vacuum on the various measurement processes that form the basis of the PV approach to general relativity. We begin by examining a constraint imposed by observation. An appropriate starting point is the expression for the fine structure constant,

$$a = \frac{e^2}{4p\, e_O \hbar c} \ \text{, where } \ c = \frac{1}{\sqrt{m_O e_O}} \ . \tag{3}$$

By the conservation of charge for elementary particles, and the conservation of angular momentum for a circularly polarized photon propagating through the vacuum (even with variable polarizability), e and $\hbar$ can be taken as constants. Given that e_O can be expected with a variable vacuum polarizability to change to $e(K) = Ke_O$, and the vacuum permeability may be a function of K, $\mu(K)$, $c(K) = 1\big/\sqrt{\mu(K)\varepsilon(K)}$. The fine structure constant therefore takes the form

$$\alpha = \frac{e^2}{4\pi\,\varepsilon_O\,\hbar c}\sqrt{\frac{\mu(K)/\mu_O}{K}} \tag{4}$$

which is potentially a function of K.

Studies that consider the possibility of the variability of fundamental constants under varying cosmological conditions, however, require that the fine structure constant remain constant in order to satisfy the constraints of observational data [9-11]. Under this constraint we obtain from Eq. (4) $\mu(K) = K\mu_O$; thus the permittivity and permeability constants of the vacuum must change together with vacuum polarizability as[1]

$$\varepsilon_o \to \varepsilon = K\varepsilon_o \ , \qquad \mu_o \to \mu = K\mu_o \ . \tag{5}$$

As a result the velocity of light changes inversely with K in accordance with

$$c\ (=1\big/\sqrt{\mu_O\varepsilon_O}\ ) \longrightarrow c/K\ \ (=1/\sqrt{\mu\varepsilon}\ )\ . \tag{6}$$

Thus, the dielectric constant of the vacuum plays the role of a variable refractive index under conditions in which vacuum polarizability is assumed to change in response to GR-type influences. As will be shown in detail later, the PV treatment of GR effects is based on the use of equations that hold in special relativity, but with the modification that the velocity of light c in the Lorentz transformations and elsewhere is replaced by the velocity of light in a medium of variable refractive index, c/K. Expressions such as $E = mc^2$ are still valid, but now take into account that $c \rightarrow c/K$, and E and m may be functions of K, and so forth.

3.2. ENERGY IN A VACUUM OF VARIABLE POLARIZABILITY

Dicke has shown by application of a limited principle of equivalence that the energy of a system whose value is E_O in flat space $(K = 1)$ takes on the value

$$E = \frac{E_O}{\sqrt{K}} \tag{7}$$

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[1] This transformation, which maintains constant the ratio $\sqrt{\mu/\varepsilon} = \sqrt{\mu_O/\varepsilon_O}$ (the impedance of free space) is just what is required to maintain electric-to-magnetic energy ratios constant during adiabatic movement of atoms from one point to another of differing vacuum polarizability [3]. Detailed analysis shows that it is also a necessary condition in the *THeμ* formalism for an electromagnetic test body to fall in a gravitational field with a composition-independent acceleration (WEP, or weak equivalence principle, verified by Eötvös-type experiments) [4-6]. Finally, this condition must be satisfied by any metric theory of gravity, which constitutes the class of viable gravity theories.

in a region where K 1 [3]. This is due to that fact that the self-energy of a system changes in response to changes in the local vacuum polarizability, analogous to the change in the stored energy of a charged air capacitor during transport to a region of differing dielectric constant.

The energy relationship given by Eq. (7) also implies, via $E_o = m_o c^2$, which becomes $E(K) = m(K)(c/K)^2$, a corollary change in mass

$$m = m_O K^{3/2}\,, \tag{8}$$

again a consequence of the change in self-energy.

3.3. ROD AND CLOCK (METRIC) CHANGES IN A VACUUM OF VARIABLE POLARIZABILITY

Another consequence of the change in energy as a function of vacuum polarizability is a change in associated frequency processes which, by the quantum condition $E = \hbar w$ and Eq. (7), takes the form

$$w = \frac{w_O}{\sqrt{K}} \;. \tag{9}$$

This, as we shall see, is responsible for the red shift in light emitted from an atom located in a gravitational potential.

From the reciprocal of Eq. (9) we find that time intervals marked by such processes are related by

$$\Delta t = \Delta t_O \cdot \sqrt{K} \;. \tag{10}$$

Therefore, in a gravitational potential (where it will be shown that $K > 1$) the time interval between clock ticks is increased (that is, the clock runs slower) relative to a reference clock at infinity.

With regard to effects on measuring rods, we note that, for example, the radius of the ground-state Bohr orbit of a hydrogen atom

$$\Delta r_O = \frac{\hbar}{m_O c}\frac{1}{a} \tag{11}$$

becomes (with c ® c/K, m_O ® m, and a constant as discussed earlier)

$$\Delta r = \frac{\Delta r_O}{\sqrt{K}} \;. \tag{12}$$

Other measures of length such as the classical electron radius or the Compton wavelength of a particle lead to the relationship Eq. (12) as well, so this relationship is general. This dependence of fundamental length measures on the variable K indicates that the dimensions of material objects adjust in accordance with local changes in vacuum polarizability - thus there is no such thing as a perfectly rigid rod. From the standpoint of the PV approach this is the genesis of the variable metric that is of such significance in GR studies.

We are now in a position to define precisely what is meant by the label "curved space." In the vicinity of, say, a planet or star, where $K > 1$, if one were to take a ruler and measure along a radius vector R to some circular orbit, and then measure the circumference C of that orbit, one would obtain $C <$ 2ðR (as for a concave curved surface). This is a consequence of the ruler being relatively shorter during the radial measuring process (see Eq. (12)) when closer to the body where K is relatively greater, as compared to its length during the circumferential measuring process when further from the body. Such an influence on the measuring process due to induced polarizability changes in the vacuum near the body leads to the GR concept that the presence of the body "influences the metric," and correctly so.

Of special interest is the measurement of the velocity of light with "natural" (i.e., physical) rods and clocks in a gravitational potential which have become "distorted" in accordance with Eqns. (10) and (11). It is a simple exercise to show that the *measured* velocity of light obtained by the use of physical rods and clocks renormalizes from its "true" (PV) value c/K to the value c. The PV formalism therefore maintains the universal constancy of the locally *measured* velocity of light.

3.4. THE METRIC TENSOR

At this point we can make a crossover connection to the standard metric tensor concept that characterizes the conventional GR formulation. In flat space a (4-dimensional) infinitesimal interval is given by the expression

$$ds^2 = c^2 dt_O^2 - \left(dx_O^2 + dy_O^2 + dz_O^2\right) . \qquad (13)$$

If rods were rigid and clocks non-varying in their performance in regions of differing vacuum polarizability, then the above expression would hold universally. However, a dx_O-length measuring rod placed in a region where $K > 1$, for example, shrinks according to Eq. (12) to $dx = dx_O / \sqrt{K}$. Therefore, the infinitesimal length which would measure dx_O were the rod to remain rigid is now expressed in terms of the dx-length rod as $dx_O = \sqrt{K} dx$. (Such constitutes a transformation between "proper" and "coordinate" values.) With a similar argument based on Eq. (10) holding for clock rate, Eq. (13) can be written

$$ds^2 = \frac{1}{K} c^2 dt^2 - K(dx^2 + dy^2 + dz^2) . \qquad (14)$$

Therefore, the infinitesimal interval takes on the form

$$ds^2 = g_{ij} dx^i dx^j , \qquad (15)$$

where g_{ij} in the above expression defines the metric tensor, and

$$dx^0 = c\,dt,\ g_{00} = 1/K,\ g_{11} = g_{22} = g_{33} = -K,\ g_{ij} = 0\ for\ i \neq j\,. \tag{16}$$

The metric tensor in this form defines an *isotropic coordinate system*, familiar in GR studies.

4. Classical Experimental Tests of General Relativity in the PV Model

In the previous sections we have established the concept of the polarizable vacuum and the effects of polarizability changes on metric (rods and clocks) behavior. In particular, we found that metric changes can be specified in terms of a single parameter K, the dielectric constant of the vacuum. This is the basis of the PV approach to GR.

In this section we note, with the aid of expressions to be derived in detail in Section 5, how K changes in the presence of mass, and the effects generated thereby. The effects of major interest at this point comprise such classical tests of GR as the gravitational redshift, the bending of light and the advance of the perihelion of Mercury. These examples constitute a good testbed for demonstrating the techniques of the PV alternative to the conventional GR curved-space approach.

For the spherically symmetric mass distribution of a star or planet it will be shown later from the basic postulates of the PV approach that the appropriate PV expression for the vacuum dielectric constant K is given by the exponential form

$$K = e^{2GM/rc^2} = 1 + \frac{2GM}{rc^2} + \frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2 + \ldots\ldots\ , \tag{17}$$

where G is the gravitational constant, M is the mass, and r is the distance from the origin located at the center of the mass M. For comparison with expressions derived by conventional GR techniques, it is sufficient to restrict consideration to a weak-field approximation expressed by expansion of the exponential to second order as shown.

As an example of application of the PV approach to a standard experimental test of GR, we consider the case of gravitational redshift. In a gravitation-free part of space, photon emission from an excited atom takes place at some frequency w_O, uninfluenced by vacuum polarizability changes. That same emission process taking place in a gravitational field, however, will, according to Eq. (9), have its emission frequency altered (redshifted) to $w = w_O/\sqrt{K}$. With the first-order correction to K = 1 given by the first two terms in Eq. (17), emission by an atom located on the surface of a body of mass M and radius R will therefore experience a redshift by an amount

$$\frac{\Delta w}{w_O} = \frac{w - w_O}{w_O} \approx -\frac{GM}{Rc^2}, \tag{18}$$

where we take $GM/Rc^2 \ll 1$. Once emitted, the frequency of the photon remains constant during its propagation to a relatively gravitation-free part of space where its frequency can then be compared against that of local emission, and the spectral shift given by Eq. (18) observed. Measurement of the redshift of the sodium D_1 line emitted on the surface of the sun and received on earth has verified Eq. (18) to a precision of 5% [12].

Experiments carried out on the surface of the earth involving the comparison of photon frequencies at different heights have improved the accuracy of verification still further to a precision of 1% [13-14]. With the two ends of the experiment separated by a vertical height *h*, the first-order frequency shift is calculated with the aid of Eqns. (9) and (18) as

$$\frac{\Delta w}{w} \approx \frac{GM}{R^2 c^2} h \,, \tag{19}$$

where *M* and *R* are the mass and radius of the earth. This experiment required a measurement accuracy of $w/w \sim 10^{-15}$ for a height $h = 22.5$ meters. It was accomplished by the use of Mössbauer-effect measurement of the difference between γ-ray emission and absorption frequencies at the two ends of the experiment.

In similar fashion, we could consider in detail other canonical examples such as the bending of light rays or perihelion advance of planetary orbits near a mass. However, rather than treating these cases individually, we can take a more general approach.

In standard textbook treatments of the classical tests of GR one begins with the Schwarzschild metric, which in isotropic coordinates is written [15]

$$\begin{aligned} ds^2 &= g_{ij}\, dx^i dx^j \\ &= \left(\frac{1 - \frac{GM}{2rc^2}}{1 + \frac{GM}{2rc^2}} \right)^2 c^2 dt^2 - \left(1 + \frac{GM}{2rc^2} \right)^4 \left(dr^2 + r^2 dq^2 + r^2 \sin^2 q df^2 \right) . \end{aligned} \tag{20}$$

Expanding the metric tensor for small departures from flatness as a Maclaurin series in (GM/rc^2), we obtain

$$g_{00} = \left(\frac{1 - \frac{GM}{2rc^2}}{1 + \frac{GM}{2rc^2}} \right)^2 = 1 - 2\left(\frac{GM}{rc^2} \right) + 2\left(\frac{GM}{rc^2} \right)^2 - \ldots \tag{21}$$

$$g_{11} = g_{22} = g_{33} = -\left(1 + \frac{GM}{2rc^2} \right)^4 = -\left[1 + 2\left(\frac{GM}{rc^2} \right) + \ldots \right] . \tag{22}$$

Similarly, in the PV approach one begins with the exponential metric defined by Eqns. (14) - (17),

$$ds^2 = g_{ij}\,dx^i dx^j \tag{23}$$
$$= e^{-2GM/rc^2} c^2 dt^2 - e^{2GM/rc^2}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right) .$$

This, when expanded to the same order as the Schwarzschild metric tensor above for small departures from unity vacuum dielectric constant, yields

$$g_{00} = e^{-2GM/rc^2} = 1 - 2\left(\frac{GM}{rc^2}\right) + 2\left(\frac{GM}{rc^2}\right)^2 - \ldots \ , \tag{24}$$

$$g_{11} = g_{22} = g_{33} = -e^{2GM/rc^2} = -\left[1 + 2\left(\frac{GM}{rc^2}\right) + \ldots\right] \ . \tag{25}$$

Comparison of Eqns. (24) - (25) with Eqns. (21) - (22) reveals that, to the order of expansion shown, the two metric tensors are identical. Since the classical tests of GR do not require terms beyond these explicitly displayed, the agreement between theory and experiment is accounted for equally in both the conventional GR and in the alternative PV formalisms.

For a *charged* mass the Schwarzschild metric is replaced by the Reissner-Nordstrøm metric in the GR approach, while in the PV approach the exponential metric is replaced by a metric involving hyperbolic functions (see Section 6.2). Again, for the weak-field case, it can be shown that the two approaches are in precise agreement to the order shown in the charge-free case.

5. Coupled Matter-Field Equations

In the preceding section we have seen that the classical tests of GR theory can be accounted for in the PV formalism on the basis of a variable vacuum dielectric constant, *K*. To carry that out we stated without proof that the appropriate mathematical form for the variation in *K* induced by the presence of mass is an exponential form. In this section we show how the exponential form is derived from first principles, and, in the process, establish the general approach to the derivation of field equations as well as the equations for particle motion. The approach consists of following standard Lagrangian techniques as outlined, for example, in Ref. 16, but with the proviso that in our case the dielectric constant of the vacuum is treated as a variable function of time and space.

5.1. LAGRANGIAN APPROACH

The Lagrangian for a free particle is given by

$$L^p = -mc^2 \cdot \sqrt{1 - \left(\frac{v}{c}\right)^2} \ \ , \tag{26}$$

which, in the presence of a variable vacuum dielectric constant K, is modified with the aid of Eqns. (6) and (8) to read

$$L^{p} = -\frac{m_O c^2}{\sqrt{K}} \sqrt{1-\left(\frac{v}{c/K}\right)^2} \quad . \tag{27}$$

This implies a Lagrangian *density* for the particle of

$$L_d^{p} = -\frac{m_O c^2}{\sqrt{K}} \sqrt{1-\left(\frac{v}{d/K}\right)^2}\, d^3\left(\mathbf{r}-\bar{\mathbf{r}}\right) \; . \tag{28}$$

Following standard procedure, the particle Lagrangian density can be extended to the case of interaction with electromagnetic fields by inclusion of the potentials $(\Phi, \mathbf{A})$,

$$L_d^{p} = -\left(\frac{m_O c^2}{\sqrt{K}} \sqrt{1-\left(\frac{v}{c/K}\right)^2} + q\Phi - q\mathbf{A}\bullet\mathbf{v}\right) d^3\left(\mathbf{r}-\bar{\mathbf{r}}\right) \; , \tag{29}$$

The Lagrangian density for the electromagnetic fields themselves, as in the case of the particle Lagrangian, is given by the standard expression (see, e.g., Ref. 16), except that again K is treated as a variable,

$$L_d^{em} = -\frac{1}{2}\left(\frac{B^2}{K m_O} - K e_O E^2\right) \quad . \tag{30}$$

We now need a Lagrangian density for the dielectric constant variable K, which, being treated as a scalar variable, must take on the standard Lorentz-invariant form for propagational disturbances of a scalar,

$$L_d^{K} = -l\, f(K)\left[(\nabla K)^2 - \frac{1}{(c/K)^2}\left(\frac{¶K}{¶t}\right)^2\right] \; , \tag{31}$$

where *f(K)* is an arbitrary function of K. As indicated by Dicke in the second citation of Ref. 3, a correct match to experiment requires that we take $l = c^4/32ðG$ and $f(K) = 1/K^2$; thus,

$$L_d^{K} = -\frac{l}{K^2}\left[(\nabla K)^2 - \frac{1}{(c/K)^2}\left(\frac{¶K}{¶t}\right)^2\right] \; . \tag{32}$$

We can now write down the total Lagrangian density for matter-field interactions in a vacuum of variable dielectric constant,

$$L_d = -\left(\frac{m_O c^2}{\sqrt{K}} \cdot \sqrt{1 - \left(\frac{v}{c/K} \right)^2} + q\Phi - q\mathbf{A} \bullet \mathbf{v} \right) d^3 (\mathbf{r} - \bar{\mathbf{r}}) - \frac{1}{2} \left(\frac{B^2}{K m_O} - K e_O E^2 \right)$$
$$- \frac{l}{K^2} \left[(\nabla K)^2 - \frac{1}{(c/K)^2} \left(\frac{\P K}{\P t} \right)^2 \right] . \tag{33}$$

5.2. GENERAL MATTER-FIELD EQUATIONS

Variation of the Lagrangian density $d \int L_d dxdydzdt$ with regard to the particle variables leads to the equation for particle motion in a variable dielectric vacuum,

$$\frac{d}{dt} \left[\frac{\left(m_O K^{3/2} \right) \mathbf{v}}{\sqrt{1 - \left(\frac{v}{c/K} \right)^2}} \right] = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \frac{\left(m_O c^2 / \sqrt{K} \right)}{\sqrt{1 - \left(\frac{v}{c/K} \right)^2}} \left(\frac{1 + \left(\frac{v}{c/K} \right)^2}{2} \right) \frac{\nabla K}{K} . \tag{34}$$

We see that accompanying the usual Lorentz force is an additional dielectric force proportional to the gradient of the vacuum dielectric constant. This term is equally effective with regard to both charged and neutral particles and accounts for the familiar gravitational potential, whether Newtonian in form or taken to higher order to account for GR effects.[2]

Variation of the Lagrangian density with regard to the K variable leads to an equation for the generation of GR vacuum polarization effects due to the presence of matter and fields. (In the final expression we use $\nabla^2 K = (1/2K)(\nabla K)^2 + 2\sqrt{K}\nabla^2\sqrt{K}$ to obtain a form convenient for the following discussion.)

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[2] Of passing interest is the fact that as m_o ® 0, but v ® c/K, the deflection of a zero-mass particle (e.g., a photon) in a gravitational field is twice that of a slow-moving particle $(v \to 0)$, an important result in GR dynamics.

$$\nabla^2\sqrt{K} - \frac{1}{(c/K)^2}\frac{\P^2\sqrt{K}}{\P t^2} = -\frac{\sqrt{K}}{4l}\left\{\frac{\left(m_O c^2/\sqrt{K}\right)}{\sqrt{1-\left(\frac{v}{c/K}\right)^2}}\left[\frac{1+\left(\frac{v}{c/K}\right)^2}{2}\right]d^3\left(\mathbf{r}-\bar{\mathbf{r}}\right)\right. \tag{35}$$

$$\left. + \frac{1}{2}\left(\frac{B^2}{Km_O} + Ke_O E^2\right) - \frac{l}{K^2}\left[(\nabla K)^2 + \frac{1}{(dK)^2}\left(\frac{\P K}{\P t}\right)^2\right]\right\}.$$

Thus we see that changes in the vacuum dielectric constant K are driven by mass density (first term), EM energy density (second term), and the vacuum polarization energy density itself (third term). The fact that the latter term corresponds to the energy density of the K variable can be seen by the following argument. We start with the Lagrangian density Eq. (32), define the momentum density by $p = \P L_d^K \big/ \P(\P K/\P t)$, and form the Hamiltonian energy density to obtain

$$H_d^K = p\left(\frac{\P K}{\P t}\right) - L_d^K = \frac{l}{K^2}\left[(\nabla K)^2 + \frac{1}{(c/K)^2}\left(\frac{\P K}{\P t}\right)^2\right]. \tag{36}$$

Eqns. (34) and (35), together with Maxwell's equations for propagation in a medium with variable dielectric constant, thus constitute the master equations to be used in discussing matter-field interactions in a vacuum of variable dielectric constant as required in the PV formulation of GR.

6. Static Field Solutions

We demonstrate application of field Eq. (35) to two static field cases with spherical symmetry: derivation of the expression introduced earlier for the gravitational field alone, and derivation of the corresponding expression for charged masses.

6.1. STATIC FIELDS (GRAVITATIONAL)

In space surrounding an uncharged spherical mass distribution the static solution *(K/ t = 0)* to Eq. (35) is found by solving

$$\nabla^2\sqrt{K} = \frac{1}{4K^{3/2}}(\nabla K)^2 = \frac{1}{\sqrt{K}}\left(\nabla\sqrt{K}\right)^2 \tag{37a}$$

or

$$\frac{d^2\sqrt{K}}{dr^2} + \frac{2}{r}\frac{d\sqrt{K}}{dr} = \frac{1}{\sqrt{K}}\left(\frac{d\sqrt{K}}{dr}\right)^2 \;, \tag{37b}$$

where we have used $(\nabla K)^2 = 4K\left(\nabla\sqrt{K}\right)^2$.

The solution that satisfies the Newtonian limit is given by

$$\sqrt{K} = e^{GM/rc^2} \tag{38a}$$

or

$$K = e^{2GM/rc^2} = 1 + 2\left(\frac{GM}{rc^2}\right) + \ldots \quad , \tag{38b}$$

which can be verified by substitution into the equation for particle motion, Eq. (34). We have thus derived from first principles the exponential form of the variable dielectric constant in the vicinity of a mass as used in earlier sections. As indicated in Section 4, this solution reproduces to appropriate order the standard GR Schwarzschild metric predictions as they apply to the weak-field conditions prevailing in the solar system.

6.2. STATIC FIELDS (GRAVITATIONAL PLUS ELECTRICAL)

For the case of a mass M with charge Q we first write the electric field appropriate to a charged mass imbedded in a variable-dielectric-constant medium,

$$\int \vec{D} \bullet \vec{da} = Ke_O E\, 4pr^2 = Q\ . \tag{39a}$$

Substitution into Eq. (35) yields (for spherical symmetry)

$$\frac{d^2\sqrt{K}}{dr^2} + \frac{2}{r}\frac{d\sqrt{K}}{dr} = \frac{1}{\sqrt{K}}\left[\left(\frac{d\sqrt{K}}{dr}\right)^2 - \frac{b^2}{r^4}\right]\ , \tag{39b}$$

where $b^2 = Q^2G/4ðe_Oc^4$.

The solution to Eq. (39) as a function of charge (represented by b) and mass (represented by $a = GM/c^2$) is given below. Substitution into Eq. (34) verifies that as r ® this expression asymptotically approaches the standard flat-space equations for particle motion about a body of charge Q and mass M.

$$\sqrt{K} = \cosh\left(\frac{\sqrt{a^2-b^2}}{r}\right) + \frac{a}{\sqrt{a^2-b^2}}\sinh\left(\frac{\sqrt{a^2-b^2}}{r}\right)\ , \quad a^2 > b^2\ . \tag{40}$$

(For $b^2 > a^2$ the solution is trigonometric.)

As noted earlier (Section 4), for the weak-field case the above reproduces the familiar Reissner-Nordstrφm metric [17].

7. Strong-Field Tests

As noted in the Abstract, both the conventional and PV approaches to GR problems lead to the same results for small departures from flatness. For increasingly larger departures from flatness, however, the two approaches, although initially following similar trends, begin to diverge with regard to specific magnitudes of effects. In the PV approach the solution for the static gravitational case yields a metric tensor that is exponential in form, in the conventional GR approach the somewhat more complex Schwarzschild solution. This discrepancy has shown up previously in other general curved-space approaches to GR as well.[3]

7.1. ASTROPHYSICAL TESTS

A major difference between the Schwarzschild (GR) and exponential (PV) metrics is that the former contains an event horizon at $R = 2GM/c^2$ which prevents radially-directed photons from escaping ("black holes"), whereas the latter has no such discontinuity (only increasingly "dark gray holes"). One consequence is that whereas the Schwarzschild solution limits neutron stars (or neutron star mergers) to ~2.8 solar masses ($2.8M_\odot$) because of black hole formation, no such constraint exists for the exponential metric. This raises the possibility that such anomalous observations as the enormous radiative output ($\sim 2M_\odot c^2$, if isotropic) of the gamma ray burster GRB990123 [20] might be interpreted as being associated with collapse of a *very* massive star (hypernova), or the collision of two high-density neutron stars [21]. The collection of additional astrophysical evidence of this and related genres would be useful in the search for discriminants between the standard GR and alternative PV approaches.

7.2. LABORATORY TESTS

For small departures from flatness it is useful to express the generalized metric in terms of the PPN (parametrized post-Newtonian) form

$$g_{00} = \left[1 - 2af + 2bf^2 - ...\right] \ , \tag{41}$$

$$g_{11} = g_{22} = g_{33} = -\left[1 + 2gf + 2df^2 + ...\right] \ , \tag{42}$$

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[3] Of special interest is the so-called Einstein-Yilmaz tensor form, in which Einstein's equations are modified by inclusion of the stress-energy tensor of the gravitational field itself on the R.H.S. of the equations, in addition to the usual matter/field stress-energy [18]. The Yilmaz modification yields exponential solutions in the form derived here by means of the PV approach. The Einstein-Yilmaz equations satisfy the standard experimental tests of GR, as well as addressing a number of mathematical issues of concern to general relativists, and are thus under study as a potentially viable modification to the original Einstein form [19].

where $f = GM/rc^2$ and a, b, g and d comprise the PPN parameters. For the case of a central mass, both the conventional Schwarzschild and PV-derived exponential solutions require by virtue of the classical tests of GR that $a = b = g = 1$. There exists, however, a predicted discrepancy with regard to the fourth parameter, d, which is

$$d = \begin{cases} 3/4 & (\text{GR Schwarzschild solution}) \\ 1 & (\text{PV exponential solution}) \end{cases} . \tag{43}$$

As detailed in Ref. [18], an argument has been put forward that the isotropy-of-mass experiments of Hughes et al. [22] and by Drever [23], and the neutron phase-shift measurements of Collela et al. [24] yield a value $d \sim 1 \pm 10^{-3}$. The data, analysis, and interpretation of such experiments provide yet further opportunities for discriminants between the standard GR and the alternative PV approaches.

8. Discussion

In overview, we have shown that a convenient methodology for investigating general relativistic (GR) effects in a non-abstract formalism is provided by the so-called polarizable-vacuum (PV) representation of GR. The PV approach treats metric perturbation in terms of a vacuum dielectric function K that tracks changes in the effective permittivity and permeability constants of the vacuum, a *metric engineering* approach, so to speak [25]. The structure of the approach is along the lines of the *THem* formalism used in comparative studies of gravitational theories.

The PV-derived matter-field Eqns. (34)-(35) are in principle applicable to a wide variety of problems. This short exposition, covering but the Schwarzschild and Reissner-Nordstrøm metrics and experimental tests of GR, is therefore clearly not exhaustive. Consideration was confined to cases of spherical symmetry and static sources,[4] and important topics such as gravitational radiation and frame-dragging effects were not addressed. Therefore, further exploration and extension of the PV approach to specific problems of interest is encouraged, again with cross-referencing of PV-derived results to those obtained by conventional GR techniques to ensure that the PV approach does not generate incomplete or spurious results.

With regard to the epistemology underlying the polarizable-vacuum (PV) approach as compared with the standard GR approach, one rather unconventional viewpoint is that expressed by Atkinson who carried out a study comparing the two [26]. "It is possible, on the one hand, to postulate that the velocity of light is a universal constant, to define 'natural' clocks and measuring rods as the standards by which space and time are to be judged, and then to discover from measurement that space-time, and space itself, are 'really' non-Euclidean; alternatively, one can *define* space as Euclidean and time as the same everywhere, and discover (from exactly the same measurements) how

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[4] However, it is known that the PV-related *THem* approach is sufficiently general that results obtained for spherically symmetric gravitational fields can be generalized to hold for nonsymmetric conditions as well.

the velocity of light, and natural clocks, rods, and particle inertias 'really' behave in the neighborhood of large masses. There is just as much (or as little) content for the word 'really' in the one approach as in the other; provided that each is self-consistent, the ultimate appeal is only to convenience and fruitfulness, and even 'convenience' may be largely a matter of personal taste..."

On the other hand, from the standpoint of what is actually measured with physical rods and clocks, the conventional tensor approach captures such measurements in a concise, mathematically self-consistent formalism (the tensor approach). Therefore, the standard approach is more closely aligned with the positivist viewpoint that underlies modern scientific thought. Nonetheless, the PV model, with its intuitive, physical appeal, can be useful in bridging the gap between flat-space Newtonian physics and the curved-spacetime formalisms of general relativity.

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10. References

1. Misner, C.W., Thorne, K.S., and Wheeler, J.A. (1973) *Gravitation*, Freeman, San Francisco, p. 5.

2. Wilson, H.A. (1921) An electromagnetic theory of gravitation, *Phys. Rev.* **17**, 54-59.

3. Dicke, R.H. (1957) Gravitation without a principle of equivalence, *Rev. Mod. Phys.* **29**, 363-376. See also Dicke, R.H. (1961) Mach's principle and equivalence, in C. Møller (ed.), *Proc. of the Intern'l School of Physics "Enrico Fermi" Course XX, Evidence for Gravitational Theories*, Academic Press, New York, pp.1-49.

4. Lightman, A.P., and Lee, D.L. (1973) Restricted proof that the weak equivalence principle implies the Einstein equivalence principle, *Phys. Rev. D* **8**, 364-376.

5. Will, C.M. (1974) Gravitational red-shift measurements as tests of nonmetric theories of gravity, *Phys. Rev. D* **10**, 2330-2337.

6. Haugan, M.P., and Will, C.M. (1977) Principles of equivalence, Eötvös experiments, and gravitational red-shift experiments: The free fall of electromagnetic systems to post-post-Coulombian order, *Phys. Rev. D* **15**, 2711-2720.

7. Volkov, A.M., Izmest'ev, A.A., and Skrotskii, G.V. (1971) The propagation of electromagnetic waves in a Riemannian space, *Sov. Phys. JETP* **32**, 686-689.

8. Heitler, W. (1954) *The Quantum Theory of Radiation*, 3rd ed., Oxford University Press, London, p. 113.

9. Alpher, R.A. (Jan.-Feb. 1973) Large numbers, cosmology, and Gamow, *Am. Sci.* **61**, 52-58.

10. Harrison, E.R. (Dec. 1972) The cosmic numbers, *Phys. Today* **25**, 30-34.

11. Webb, J.K., Flambaum, V.V., Churchill, C.W., Drinkwater, M.J., and Barrow, J.D. (1999) Search for time variation of the fine structure constant, *Phys. Rev. Lett.* **82**, 884-887.

12. Brault, J.W. (1963) Gravitational red shift of solar lines, *Bull. Amer. Phys. Soc.* **8**, 28.

13. Pound, R.V., and Rebka, G.A. (1960) Apparent weight of photons, *Phys. Rev. Lett.* **4**, 337-341.

14. Pound, R.V., and Snider, J.L. (1965) Effect of gravity on nuclear resonance, *Phys. Rev. Lett.* **13**, 539-540.

15. Ref. 1, p. 840.

16. Goldstein, H. (1957) *Classical Mechanics*, Addison-Wesley, Reading MA, pp. 206-207.

17. Ref. 1, p. 841.

18. Mizobuchi, Y. (1985) New theory of space-time and gravitation - Yilmaz's approach, *Hadronic Jour.* **8**, 193-219.

19. Alley, C.O. (1995) The Yilmaz theory of gravity and its compatibility with quantum theory, in D.M. Greenberger and A. Zeilinger (eds.), *Fundamental Problems in Quantum Theory: A Conference Held in Honor of Professor John A. Wheeler*, Vol. 755 of the Annals of the New York Academy of Sciences, New York, pp. 464-475.

20. Schilling, G. (1999) Watching the universe's second biggest bang, *Science* **283**, 2003-2004.

21. Robertson, S.L. (1999) Bigger bursts from merging neutron stars, *Astrophys. Jour.* **517**, L117-L119.

22. Hughes, V.W., Robinson, H.G. and Beltran-Lopez, V. (1960) Upper limit for the anisotropy of inertial mass from nuclear resonance experiments, *Phys. Rev. Lett.* **4**, 342-344.

23. Drever, R.W.P. (1961) A search for anisotropy of inertial mass using a free precession technique, *Phil. Mag.* **6**, 683-687.

24. Collela, R. Overhauser, A.W., and Werner, S.A. (1975) Observation of gravitationally induced quantum mechanics, *Phys. Rev Lett.* **34**, 1472-1474.

25. Puthoff, H.E. (1996) SETI, the velocity-of-light limitation, and the Alcubierre warp drive: an integrating overview, *Physics Essays* **9**, 156-158.

26. Atkinson, R. d'E. (1962) General relativity in Euclidean terms, *Proc. Roy. Soc.* **272**, 60-78.