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A Hamilton-like vector for the special-relativistic Coulomb problem. (English summary)

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In the non-relativistic central force problem with potential $1/r$, and with radiation ignored, the Hamilton vector $\mathbf{h} = \mathbf{v} - (\mathbf{k}/\mathbf{L})\widehat{\theta}$ (which is simply related to the Laplace-Runge-Lenz vector) is a conserved 3-component quantity in addition to the energy and angular momentum. Correspondingly its use simplifies solution of the non-relativistic Kepler and Coulomb problems, though the method does not automatically extend to the relativistic case. Here the authors present a generalised Hamilton 3-vector $\mathbf{h} = \gamma(\mathbf{v} - (k/L)\widehat{\theta})$ that similarly simplifies the relativistic problem—even though this $\mathbf{h}$ is not conserved—using it to obtain an efficient solution for the relativistic Coulomb problem (still neglecting radiation). They give a complete catalogue of the 6 different cases, with illustrations of example trajectories for both the bound ($E < mc^2$) and unbound ($E \geq mc^2$) sub-cases of critical, sub-critical, and super-critical angular momenta. There is a short discussion of the history of the subject without dwelling on the $\mathrm{SO}(4)$ symmetry of the non-relativistic problem, which is destroyed in the relativistic case.

Reviewed by *Michael Ibison*