A Massless Classical Electron

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Abstract

This paper explores the response to arbitrary electromagnetic fields of a classical charge with no intrinsic mechanical mass. It is argued that such a particle is feasible only if free of self action, which is achieved here by adopting the direct action version of electromagnetism. A general equation of motion is then found in terms of the external fields that permits super-luminal speeds and time-reversals. The outcome is a realization of the Feynman and Stueckleberg conjecture that electrons and positrons are different segments on a common trajectory. Some suggestions are made for further development, including the acquisition of mass through electromagnetic interaction, and exploration of the relation to QM.

I. INTRODUCTION

A. Motivation

Stueckleberg [1], [2] and Feynman [3], suggested that all electrons and positrons possess the same qualities because they are the same particle – traveling in opposite directions in time. Although the idea is readily interpretable in classical electrodynamics (CED), the traditional formulation prohibits superluminal speeds and time-reversals, and apparently, therefore, cannot endorse the Feynman-Stueckleberg conjecture. That conclusion, however, applies only to charged particles having an intrinsic (mechanical) rest mass, whereas CED without the traditional inertial-mass action turns out to permit well-defined superluminal and time-reversing trajectories, and so offers an opportunity to explore the idea further.

Since *observed* charges evidently have mass, the conjecture can be valid only if the observed mass is not intrinsic, but acquired through an interaction. Consequently, any discussion of the massless (bare) charge condition must be of a primative state of affairs not accessible to observation, out of which (it may be hoped) observed - massive - behavior would emerge. Attempts to explain inertia as derived from an electromagnetic interaction therefore provide some justification for this investigation. Feynman in particular appears to have been sympathetic to the idea that mass originates from an electromagnetic interaction [4]. This possibility encompasses recent efforts calling upon an enhanced role for the ZPF [5],[6],[7],[8],[9],[10],[11],[12]. However, the classical electron models of Poincaré [13] and Schwinger [14] are to be excluded because they do not admit a bare massless charge; their electrons rely upon unspecified non-electromagnetic forces to hold the particle together, and which contribute to the final (observed) mass.

B. Self action

In CED the action of the particle's own field upon itself gives rise to an infinite self energy that must be compensated by asserting a negative infinite bare mass to leave a finite, observable mass. In QED the situation is made worse by additional infinities due to interaction with the transverse-polarized vacuum field (the ZPF). In both cases, a literal interpretation of the mathematics leads to the conclusion that the observed mass is really a delicately balanced difference between the electromagnetic and mechanical energies, both of whose magnitudes are infinite. It follows that one cannot model a massless charge simply by omitting the contribution to the action from the mechanical mass - classically, the term $m_0 \int dt\sqrt{1-v^2}$ - one must at the same time deal with the infinite self action.

The method explored here is simply to deny self action altogether, and adopt the direct action version of CED. This presentation of electrodynamics originated with Schwarzschild, Tetrode, and Fokker in the early part of the last century [15],[16],[17], though it lacked an explanation for exclusively retarded radiation and the radiation reaction on the source. Dirac [18], showed that radiation reaction arises if the advanced fields are set to zero, for which the Wheeler and Feynman [19], [20] absorber theory gave a physical justification. Intrinsic to this presentation of direct action EM is a dynamics written in terms of many time parameters, one for each particle degree of freedom. It is to be distinguished from a separate line of development characterized by a single time parameter for the whole system, necessary for a Hamiltonian description of the dynamics. An exposition of this latter approach is given by Trump and Schieve [21]. Comparative reviews of the two approaches are given by Hill [22], and by Kerner [23]. In both forms of direct action electrodynamics, EM fields, if they are used at all, are purely mathematical devices for conveying 'interaction' between pairs of charged particles; there are no independent (vacuum) fields. Classically and quantum-mechanically there is then no self energy at all. And, because there there are no vacuum fields, quantum-electrodynamics no longer suffers from ZPF-induced infinities [24],[25],[26],[27].

The particular implementation of direct action EM advocated by Wheeler and Feynman [19], [20] is contingent on the existence of relatively cold distant absorbers of radiation on the future light cone, invoked in order to explain the perceived predominance of exclusively retarded radiation. These absorbers provide a sink for radiation, permitting radiation energy and momentum to flow away from local sources, thereby performing the role traditionally played by the vacuum degrees of freedom. And the flow of radiation energy is properly accompanied by a reaction back upon the source, performing the role traditionally played by the self-fields. The time-asymmetric boundary condition (cold absorbers on the future light cone) also succeeds in explaining how time-asymmetric radiation arises in an intrinsically time-symmetric theory. (The equations of classical electrodynamics, with or without field degrees of freedom, are of course intrinsically time-symmetric.) But the absorber theory places tight constraints on the cosmological expansion that must agree with observation [28]. And although this investigation of the behavior of massless charges is not committed to the absorber theory, if not that, then some other satisfactory explanation for the apparent predominance of retarded radiation must be found: Direct action EM admits no radiation reaction, and therefore no exchange of energy or momentum with the external fields. Therefore some symmetry-breaking mechanism must come to the fore at the same time that the bare charge acquires mass from its alleged interaction.

Distinct from the distant absorber work of Wheeler and Feynman, here the charge sources will not be ascribed an intrinsic mechanical mass; the focus of this document is on the behavior of classical charges in their allegedly pre-mass condition. Detailed discussion of a few strategies for the emergence of mass in a classical framework is left as a topic for discussion elsewhere, though brief suggestions are offered in the hope of conferring some justification for this investigation as part of a broader program of effort.

Broadly then, the investigation here starts with the simplest possible version of classical electrodynamics: with the particle stripped of mechanical mass and radiation reaction, and the fields stripped of vacuum degrees of freedom and time-asymmetry. It is hypothesized that these qualities will emerge upon assembly and interaction of multiple charges constrained by particular boundary conditions.

II. EQUATION OF MOTION

A. Action and the Euler equation

The following uses Heaviside-Lorentz units with $c = 1$ and the convention $u^a v_a = u_0 v_0 - \mathbf{u} \cdot \mathbf{v}$.

The contribution to the action from a single massless charge, assuming the fields are given, is just

$$I = -\int d^4 y A_\mu (y) \, j^\mu (y) = -e \int d\lambda A_\mu (x(\lambda)) \, u^\mu (\lambda) \tag{1}$$

where we have used that the 4-current due to a single charge is

$$j^\mu (y) = e \int d\lambda u^\mu (\lambda) \, \delta^4 (y - x(\lambda)) \, ; \quad u^\mu (\lambda) \equiv \frac{dx^\mu (\lambda)}{d\lambda} \tag{2}$$

where λ is any ordinal parameterization of the trajectory, and where x and y are 4-vectors. From its definition, the current is divergenceless for as long as the trajectory has no visible end-points. In the event that it is necessary to refer to other particles, let the particular source that is the subject of Eq.(1) have label l. And consistent with the maxim of direct action without self action, the potential in Eq.(1) must be that of other sources, which therefore can be written

$$A^{\mu}_{(\bar{l})} = \sum_{\substack{k=1\\k\neq l}} A^{\mu}_{(k)}; \quad A^{\mu}_{(k)} = G * j^{\mu}_{(k)} \tag{3}$$

where the bar over the l signifies that the potential is formed from contributions from all sources except the l^{th} source; where G is the time-symmetric Green's function for the wave equation; and the $*$ represents convolution

$$A^{\mu}_{(k)}(y) = \frac{1}{4\pi}\int d^4x\delta\left((y-x)^2\right) j^{\mu}_{(k)}(x) = \frac{e_{(k)}}{4\pi}\int d\kappa\delta\left(\left(y - x_{(k)}(\kappa)\right)^2\right) u^{\mu}_{(k)}(\kappa), \tag{4}$$

where $(y-x)^2 \equiv (y-x)^{\mu}(y-x)_{\mu}$. These 'particle-specific' fields are the same as those by Leiter [29]. Putting this into Eq.(3) and putting that into Eq.(1) and then summing over l would cause each unique product (i.e. pair) of currents to appear exactly twice. Considering every particle position as an independent degree of freedom, the resulting total action, consistent with the action Eq.(1) for just one degree of freedom, is, therefore

$$I_{all} = -\frac{1}{8\pi}\sum_{\substack{k,l\\k\neq l}} e_{(k)}e_{(l)}\int d\kappa\int d\lambda\delta\left(\left(x_{(k)}(\kappa) - x_{(l)}(\lambda)\right)^2\right) u^{\mu}_{(k)}(\kappa)\, u_{\mu(l)}(\lambda). \tag{5}$$

Equivalent to the supposition that the potential from the other sources, $k \neq l$, is given, is that the fields are in no way correlated with the motion of the single source responsible for the current j in Eq.(1). Therefore this investigation may be regarded as an analysis of the state of affairs pertaining to the first of an infinite sequence of iterations of the interaction between the l^{th} current and the distant $k \neq l$ currents responsible for the fields.

With the fields given, the Euler equation for the (massless) lone particle degree of freedom in Eq.(1) is simply that the Lorentz force on the particle in question must vanish:

$$F^{\nu\mu}_{(\bar{l})} u_{\mu(l)} = 0 \tag{6}$$

where F is the EM field-strength tensor, wherein the fields $\mathbf{E}$ and $\mathbf{B}$ are to be evaluated along the trajectory. In 3+1 form, and omitting the particle labels, this is

$$\frac{dt(\lambda)}{d\lambda}\mathbf{E}\left(\mathbf{x}(\lambda), t(\lambda)\right) + \frac{d\mathbf{x}(\lambda)}{d\lambda} \times \mathbf{B}\left(\mathbf{x}(\lambda), t(\lambda)\right) = \mathbf{0} \tag{7}$$

where $\mathbf{E}$ and $\mathbf{B}$ can be found from the usual relations to A.

B. A nodal surface constraint

For Eq.(6) to have a solution, the determinant of F must vanish, which gives:

$$\mathbf{E} \cdot \mathbf{B} = 0 \tag{8}$$

This condition imposes a constraint on the values of the fields on the trajectory and therefore, if the fields are given, on the set of possible paths that a trajectory can take. Eq.(8) is consistent with the condition that the Lorentz force on the particle must vanish since it is the well-known constraint on the fields such that there exist a frame in which the electric field is zero. Hence, in an environment of arbitrary field variation, Eq.(8) selects the surface upon which a charge source may conceivably see no electric field in its own frame. It is possible to regard Eq.(8) as a constraint on the initial conditions, i.e. the (spatial) placement of the charge source at some historical time, rather than a constraint on the fields: Only if the source is initially placed upon this surface, will a trajectory exist (see below) consistent with the presumption of masslessness and consequent vanishing of the Lorentz force.

It is interesting to ask about the topology of the surface, and in particular if the fields can be so arranged that the points solving Eq.(8) generate multiple unconnected surfaces, which might connote localization of the particle. This question has not been answered to date, though it is conceivable that it will turn out to be of little practical import once the loop is closed and particle's own fields are permitted to act upon the distant sources.

Since Eq.(8) is required to be true for all λ-time along the trajectory, it must be true that all the derivatives with respect to λ of the function $S(\mathbf{x}, t) \equiv \mathbf{E}(\mathbf{x}, t) \cdot \mathbf{B}(\mathbf{x}, t)$ are zero. In order to solve Eq.(6), we will need, in particular, that

$$\frac{dS}{d\lambda} = u^{\mu} \partial_{\mu} S = 0, \tag{9}$$

which just says that if the particle is to remain on the surface, u^{μ} must be orthogonal to the surface 4-normal.

C. Solution for the trajectory in terms of the fields

Writing Eq.(7) in the form

$$\dot{t}\mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B} = \mathbf{0} \tag{10}$$

where dots indicate differentiation with respect to λ, it may be observed that $\dot{\mathbf{x}}\cdot\mathbf{E}=0$; the velocity is always perpendicular to the local electric field. The vectors $\mathbf{E}$ and $\mathbf{B}$ are mutually orthogonal and both of them are orthogonal to $\mathbf{E}$ because $\mathbf{E}\cdot\mathbf{B}=0$. Therefore they can serve as an orthogonal basis for the velocity:

$$\dot{\mathbf{x}}=\alpha\mathbf{E}\times\mathbf{B}+\beta\mathbf{B} \tag{11}$$

where α and β are undetermined coefficients. Substitution of this expression into Eq.(10) gives

$$\begin{gathered}\mathbf{0}=\dot{t}\mathbf{E}+\alpha\left(\mathbf{E}\times\mathbf{B}\right)\times\mathbf{B}=\dot{t}\mathbf{E}+\alpha\left(\left(\mathbf{E}\cdot\mathbf{B}\right)\mathbf{B}-B^2\mathbf{E}\right)\\ \Rightarrow\alpha=\dot{t}/B^2\end{gathered} \tag{12}$$

unless perhaps $\mathbf{B}$ is zero. Substitution of Eqs.(11) and (12) into Eq.(9) then gives

$$\begin{aligned}&\dot{t}\frac{\partial S}{\partial t}+\frac{\dot{t}}{B^2}\left(\mathbf{E}\times\mathbf{B}\right)\cdot\nabla S+\beta\mathbf{B}\cdot\nabla S=0\\ &\Rightarrow\beta=-\frac{\dot{t}}{B^2\mathbf{B}\cdot\nabla S}\left(\left(\mathbf{E}\times\mathbf{B}\right)\cdot\nabla S+B^2\frac{\partial S}{\partial t}\right)\end{aligned} \tag{13}$$

unless perhaps $\mathbf{B}\cdot\nabla S$ is zero. With Eqs. (12) and (13), the velocity, Eq.(11), is

$$\begin{aligned}\dot{\mathbf{x}}&=\frac{\dot{t}}{B^2\mathbf{B}\cdot\nabla S}\left(\left(\mathbf{B}\cdot\nabla S\right)\mathbf{E}\times\mathbf{B}-\left(\left(\mathbf{E}\times\mathbf{B}\right)\cdot\nabla S+B^2\frac{\partial S}{\partial t}\right)\mathbf{B}\right)\\ &=\frac{\dot{t}}{B^2\mathbf{B}\cdot\nabla S}\left(\left(\mathbf{B}\times\left(\mathbf{E}\times\mathbf{B}\right)\right)\times\nabla S-B^2\mathbf{B}\frac{\partial S}{\partial t}\right)\\ &=\frac{\dot{t}}{B^2\mathbf{B}\cdot\nabla S}\left(\left(B^2\mathbf{E}-\left(\mathbf{E}\cdot\mathbf{B}\right)\mathbf{B}\right)\times\nabla S-B^2\mathbf{B}\frac{\partial S}{\partial t}\right)\\ &=\frac{\dot{t}}{\mathbf{B}\cdot\nabla S}\left(\mathbf{E}\times\nabla S-\mathbf{B}\frac{\partial S}{\partial t}\right).\end{aligned} \tag{14}$$

Therefore the 4-velocity is

$$u^\mu=f\left(x(\lambda),\lambda\right)w^\mu;\qquad w^\mu\equiv-\widetilde{F}^{\mu\nu}\partial_\nu S=\left(\mathbf{B}\cdot\nabla S,\mathbf{E}\times\nabla S-\mathbf{B}\frac{\partial S}{\partial t}\right) \tag{15}$$

(we will use the convention that a 4-vector with a non-repeated symbolic - non-numerical - index, e.g. u^μ, means the set of 4 coordinates, rather than a single element). Here $\widetilde{F}$ is the dual of F ($\widetilde{F}^{ab}=\epsilon^{abcd}F_{cd}$ where ϵ is the totally anti-symmetric tensor [31]) and where, because $\dot{t}$ is not known,

$$f\left(x(\lambda),\lambda\right)=\dot{t}/\mathbf{B}\cdot\nabla S \tag{16}$$

is an arbitrary function, undetermined by Eq.(10). That $u^{\mu} = -f\left(x\left(\lambda\right),\lambda\right)\widetilde{F}_{\mu\kappa}\partial^{\kappa}S$ solves Eq.(6) is confirmed upon substitution:

$$F^{\nu\mu}u_{\mu} = -fF^{\nu\mu}\widetilde{F}_{\mu\kappa}\partial^{\kappa}S. \tag{17}$$

But it is easily computed that

$$F^{\nu\mu}\widetilde{F}_{\mu\kappa} = \delta^{\nu}_{\kappa}S, \tag{18}$$

so Eq.(17) is

$$F^{\nu\mu}u_{\mu} = -fS\partial^{\nu}S \tag{19}$$

which is zero on $S = 0$, as required.

D. The trajectory as a sequence of 4-vectors

Shortly it will be seen that the massless charge can move at superluminal speeds. Use of the phrase 'Lorentz invariance' is to be understood to be limited to *sub-luminal* boosts of the *reference frame.*

One obtains from Eq.(14) that

$$\mathbf{v}\left(\mathbf{x},t\right) = \frac{d\mathbf{x}/d\lambda}{dt/d\lambda} = \frac{\mathbf{E}\times\nabla S - \mathbf{B}\partial S/\partial t}{\mathbf{B}\cdot\nabla S} \tag{20}$$

is the ordinary velocity of the trajectory passing through $(t(\lambda),\mathbf{x}(\lambda))$. The right hand side is an arbitrary function of $\mathbf{x}$ and t, decided by the fields. In general, Eq.(20) will not admit a solution of the form $\mathbf{x} = \mathbf{f}(t)$ since the solution trajectory may be non-monotonic in time, demanding, instead, a parametric description. With this caveat, in principle Eq.(20) may be solved to give the trajectory, and is therefore a complete description for a single trajectory, as it stands, provided one ignores the *sense* (see below). Let us suppose for now that the trajectory is sparse, so that u^{μ} defined in Eq.(15) cannot be a 4-vector field, because it is not defined off the trajectory. Then one would like to parameterize the trajectory in a Lorentz invariant way, so that u along the trajectory is a (Lorentz) 4-vector. This requires that the norm

$$u^{\mu}u_{\mu} = f^{2}\left(x(\lambda),\lambda\right)w^{\mu}\left(x(\lambda)\right)w_{\mu}\left(x(\lambda)\right) \tag{21}$$

is a constant scalar (i.e. a 4-scalar), where

$$w^{\mu}w_{\mu} = \widetilde{F}^{\mu\nu}\widetilde{F}_{\mu\lambda}\left(\partial_{\nu}S\right)\left(\partial^{\lambda}S\right) = \left(\mathbf{B}\cdot\nabla S\right)^{2} - \left(\mathbf{E}\times\nabla S - \mathbf{B}\partial S/\partial t\right)^{2}. \tag{22}$$

Then it is clear from Eq.(21) that (up to an arbitrary universal constant) that one must set the arbitrary function f to

$$f\left(x(\lambda),\lambda\right)=\frac{\sigma}{\sqrt{|w^{\mu}w_{\mu}|}} \tag{23}$$

where $\sigma=\pm 1$. Then the norm is

$$u^{\mu}u_{\mu}=\text{sign}\left(w^{\mu}w_{\mu}\right)=\text{sign}\left(1-v^{2}\right) \tag{24}$$

which is 1 in the sub-luminal segments of the trajectory, and -1 in the superluminal segments. Eqs.(15) and (23) now give the desired solution for the 4-velocity in terms of the external fields:

$$\begin{aligned}\{u^{\mu}\}\equiv\left(\frac{dt}{d\lambda},\frac{d\mathbf{x}}{d\lambda}\right)&=\frac{\sigma\widetilde{F}^{\mu\nu}\partial_{\nu}S}{\sqrt{\left|\widetilde{F}^{\alpha\beta}\widetilde{F}_{\alpha\gamma}\left(\partial_{\beta}S\right)\left(\partial^{\gamma}S\right)\right|}} \\ &=\frac{\sigma\left(\mathbf{B}\cdot\nabla S,\mathbf{E}\times\nabla S-\mathbf{B}\partial S/\partial t\right)}{\sqrt{\left|\left(\mathbf{B}\cdot\nabla S\right)^{2}-\left(\mathbf{E}\times\nabla S-\mathbf{B}\partial S/\partial t\right)^{2}\right|}} \\ &=\text{sign}\left(\sigma\mathbf{B}\cdot\nabla S\right)\frac{1}{\sqrt{|1-v^{2}|}}\left(1,\mathbf{v}\right)\end{aligned} \tag{25}$$

where $\mathbf{v}$ is given by Eq.(20). From Eq.(24) one has

$$u^{\mu}u_{\mu}\equiv\left(\frac{dt}{d\lambda}\right)^{2}-\left(\frac{d\mathbf{x}}{d\lambda}\right)^{2}=\text{sign}\left(1-v^{2}\right)\Rightarrow d\lambda=|dt|\sqrt{|1-v^{2}|} \tag{26}$$

and therefore, following the particular choice Eq.(23) for f, λ is now a Lorentz invariant parameter that in the sub-luminal segments is just the commonly defined proper time τ, but generalized so that it remains an ordinal parameter throughout the trajectory. The trajectory is now divided up into a sequence of sub-luminal and superluminal segments, which designation is Lorentz invariant, and for which the norm is 1 and –1 respectively (The segment boundary points and the sub-luminal and superluminal labels are Lorentz invariant because the three conditions $v<1$, $v=1$, and $v>1$ are Lorentz invariants.) Hence, in each segment u^{μ} is now a true (Lorentz) 4-vector.

E. CPT and other invariants

With reference to Fig. 1 wherein a time reversal occurs at point Q, the segments PQ and QR have different signs for $dt/d\lambda$. However, which sign is attributed to which segment (the direction of the arrow in the figure) is not decided by Eq.(20). Instead, the sense of the trajectory must be instantiated at some point 'by hand'. Noticing that Eqs.(16) and (23) give

$$\mathrm{sign}\left(\sigma\mathbf{B}\cdot\nabla S\right)=\mathrm{sign}\left(\frac{dt}{d\lambda}\right) \tag{27}$$

it is clear that $\sigma=\pm1$ is the degree of freedom that permits one to choose the sign of just one segment, the sign of all other segments on that trajectory being decided thereafter. If there is only one trajectory then the sign of σ is a common factor for the whole action, so this choice will amount to no more than a convention without any physical consequences unless some (additional) absolute sense specificity is introduced into the dynamics. But if there are multiple, unconnected, trajectories, then clearly their relative senses will be important.

For given fields at a fixed space-time location, $(\mathbf{x},t)$, the change $\sigma\to-\sigma$ in the equation of motion Eq.(25) is equivalent to $dt\to-dt$, $d\mathbf{x}\to-d\mathbf{x}$. Consistent with CPT invariance therefore, σ can be interpreted as the sign of the charge (at some fixed point on the trajectory). Accordingly, the 4-current Eq.(2), must be redefined to relieve e therein of the role of deciding the sign of the charge:

$$j^{\mu}\left(y\right)=\left|e\right|\int d\lambda u^{\mu}\left(\lambda\right)\delta^{4}\left(y-x\left(\lambda\right)\right) \tag{28}$$

where now it is understood that $dt/d\lambda$ can take either sign. Consequently the total action Eq.(5) becomes

$$I_{all}=-\frac{e^{2}}{8\pi}\sum_{\substack{k,l\\k\neq l}}\int d\kappa\int d\lambda\delta\left(\left(x_{(k)}\left(\kappa\right)-x_{(l)}\left(\lambda\right)\right)^{2}\right)u_{(k)}^{\mu}\left(\kappa\right)u_{\mu(l)}\left(\lambda\right). \tag{29}$$

Now, in accord with the conjecture of Stueckleberg and Feynman, the alternating segments of positive and negative signs (of $\sigma\mathbf{B}\cdot\nabla S$) along the trajectory are to be regarded as denoting electrons and positrons respectively. From the perspective of uniformly increasing laboratory time t, the electrons and positrons are created and destroyed in pairs, as illustrated in Fig. 1. Note that charge is conserved in t just because these events occur in (oppositely charged) pairs as entry and exit paths to and from the turning points. (If the total trajectory is not closed, charge is not conserved at the time of the two endpoints of the whole trajectory.)

For the particular case of an anti-clockwise circular trajectory in x and t, Fig. 2 identifies the eight different segment-types corresponding to charge-type, direction in time, direction in space, and speed (sub-luminal versus superluminal). superluminal, $v>1$, segments remain superluminal when viewed from any (sub-luminally) boosted frame. Likewise, segments with $v < 1$ remain sub-luminal when viewed from any (sub-luminally) boosted frame. That is, as mentioned above, the labels $v < 1$ and $v > 1$ are Lorentz invariant. But the invariant status of these labels can be regarded as a consequence of the types of allowed transformation rather than an intrinsic (i.e. truly invariant) property of a segment of the trajectory, since it is really due to the restriction of the boost transformations to sub-luminal velocities. However, having permitted the massless particle to travel superluminally, one should be prepared to consider augmentation of the traditional set of transformations to include superluminal boosts of the frame of reference. Upon replacing the traditional γ in the Lorentz transformation formulae with $\gamma = 1/\sqrt{|1 - v^2|}$ and permitting superluminal boosts (an 'extended' Lorentz transformation), the labels $v < 1$ and $v > 1$ cease to be immutable aspects of the trajectory. The points $v = 1$, however, remain immutable.

From the form of the Lorentz transformation, the sign of the direction in time of a sub-luminal segment cannot be changed by applying a (sub-luminal) boost transformation. Therefore, the sign of the charge is a Lorentz invariant. But with reference to Fig. 2, in the labeling exterior to the circle (wherein the direction in time is always positive), the charge can change sign under a (sub-luminal) boost transformation if it is traveling superluminally. This is apparent from Fig. 1, where at the pair creation and destruction events $dt/d\lambda = 0$, whereas $d\mathbf{x}/d\lambda \neq \mathbf{0}$ implying that $v = |d\mathbf{x}/dt|$ there is infinite. And if extended Lorentz transformations are permitted, then no part of the trajectory can be given an immutable label corresponding to the sign of charge.

The hypersurface $\mathbf{E} \cdot \mathbf{B} = 0$ is a Lorentz-invariant collection of 4-points (events) arising here from the requirement that the determinant of F vanish. One might ask of the other Lorentz invariant associated with the field strengths, $E^2 - B^2$, and why not the hypersurface $E^2 - B^2 = 0$ instead? It can be inferred from the fact that the latter quantity is the determinant of $\widetilde{F}$ that the source of the broken symmetry lies in the fact of the existence of the electric charge but not magnetic charge; the trajectory of a magnetic charge, were it to exist, would be constrained to lie on the hypersurface $E^2 - B^2 = 0$.

III. DYNAMICS

A. Power flow

Whilst following the instructions of the EM field, the particle generates its own advanced and retarded secondary fields as a result of its motion as determined by the usual EM formulae. By taking the scalar product of Eq.(20) with $\mathbf{v}$, one observes that

$$\mathbf{v}\left(x(\lambda)\right)\cdot\mathbf{E}\left(x(\lambda)\right)=0 \tag{30}$$

from which it can be concluded that the massless charge cannot absorb power from the fields. (Of course, if the system were properly closed, one could not arbitrarily pre-specify the fields; the incident and secondary fields would have to be self-consistent.) The massless charge cannot absorb energy from the field because there is no internal degree of freedom wherein such energy could be 'stored'.

B. Acceleration

To facilitate a comparison between the motion of the massless charge, and the observed behavior of a massive particle classical to the Lorentz force, from Eq.(22) the proper acceleration of the massless charge, is

$$a^{\mu}=\frac{du^{\mu}}{d\lambda}=u^{\lambda}\partial_{\lambda}u^{\mu}=\frac{\widetilde{F}^{\lambda\alpha}\left(\partial_{\alpha}S\right)}{\sqrt{\left|\widetilde{F}_{\gamma\beta}\widetilde{F}^{\gamma\delta}\left(\partial^{\beta}S\right)\left(\partial_{\gamma}S\right)\right|}}\partial_{\lambda}\left(\frac{\widetilde{F}^{\mu\nu}\left(\partial_{\nu}S\right)}{\sqrt{\left|\widetilde{F}_{\pi\rho}\widetilde{F}^{\pi\sigma}\left(\partial^{\rho}S\right)\left(\partial_{\sigma}S\right)\right|}}\right) \tag{31}$$

where the factor of $\sigma^2=1$ has been omitted. So that the motion is defined, both $\mathbf{E}$ and $\mathbf{B}$ must be non-zero, or else it must be assumed that one or both must default to some noise value. One might then ask if the space part of the proper acceleration is correlated with the Lorentz force, i.e. whether

$$\mathbf{F}\cdot\mathbf{a}=\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot d\mathbf{u}/d\lambda \tag{32}$$

is non-zero. But it is recalled that the massless particle executes a path upon which the Lorentz force is always zero. Specifically, from Eq.(20),

$$\mathbf{v}\times\mathbf{B}=\frac{\left(\mathbf{E}\times\nabla S-\mathbf{B}\partial S/\partial t\right)\times\mathbf{B}}{\mathbf{B}\cdot\nabla S}=\frac{\left(\mathbf{E}\times\nabla S\right)\times\mathbf{B}}{\mathbf{B}\cdot\nabla S}=\frac{\left(\mathbf{E}\cdot\mathbf{B}\right)\nabla S-\left(\mathbf{B}\cdot\nabla S\right)\mathbf{E}}{\mathbf{B}\cdot\nabla S}=-\mathbf{E} \tag{33}$$

and therefore $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0}$, and obviously therefore, $\mathbf{F} \cdot \mathbf{a} = 0$. It is concluded that the proper 3-acceleration is always orthogonal to the applied 3-force.

C. Motion near a charge with magnetic dipole moment

As an example of a one-body problem, i.e. of a test charge in a given field, we here consider a static classical charge point charge with electric field

$$\mathbf{E} = \frac{e\hat{\mathbf{r}}}{4\pi r^2} \tag{34}$$

that is coincident with the source of a magnetic dipole field of magnitude μ oriented in the z direction:

$$\mathbf{B} = \frac{\mu\left(3\hat{\mathbf{r}}z - r\hat{\mathbf{z}}\right)}{4\pi r^4} \tag{35}$$

(see, for example, [31]). Then

$$\mathbf{E} \cdot \mathbf{B} = \frac{e\mu z}{8\pi^2 r^6}, \tag{36}$$

and so the constraint that the particle trajectory be confined to the surface $\mathbf{E} \cdot \mathbf{B} = 0$ demands that $z = 0$; i.e., the particle is confined to the equatorial plane for all time. The gradient in the plane is

$$\nabla\left(\mathbf{E} \cdot \mathbf{B}\right)|_{z=0} = \frac{e\mu\hat{\mathbf{z}}}{8\pi^2 \rho^6} \tag{37}$$

where, $\rho = \sqrt{x^2 + y^2}$. With this, and using that at $z = 0$, $\mathbf{B} = -\mu\hat{\mathbf{z}}/4\pi\rho^3$, one obtains for the denominator in Eq.(20)

$$\mathbf{B} \cdot \nabla\left(\mathbf{E} \cdot \mathbf{B}\right) = -\frac{e\mu^2}{32\pi^3 \rho^9}. \tag{38}$$

Since $\mathbf{E} \cdot \mathbf{B}$ is constant in time, the numerator in Eq.(20) is just

$$\mathbf{E} \times \nabla\left(\mathbf{E} \cdot \mathbf{B}\right) = \frac{e^2\mu}{32\pi^3\rho^8}\hat{\rho} \times \hat{\mathbf{z}} = -\frac{e^2\mu}{32\pi^3\rho^8}\hat{\phi}, \tag{39}$$

the latter being a cylindrical polar representation of the vector with basis $\left(\hat{\rho}, \hat{\phi}, \hat{\mathbf{z}}\right)$. Substitution of Eqs.(38) and (39) into Eq.(20) gives that the velocity in the cylindrical basis is

$$v_\phi \equiv \frac{d\phi}{dt}\rho = \frac{e}{\mu}\rho, \quad v_\rho = v_z = 0 \tag{40}$$

which immediately gives that

$$z = 0, \quad \rho = const, \quad \dot{\phi} = e/\mu. \tag{41}$$

So it is found that the massless charge is constrained to execute, with frequency e/μ, a circular orbit in the equatorial plane about the axis of the magnetic dipole, as illustrated in Fig. 3. The solution is determined up to two constants: the radius of the orbit, ρ, and the initial phase (angle in the x, y plane when $t = 0$). It is interesting to note that if the magnetic moment is that of the electron, i.e. $\mu = e/2\omega_c$ where ω_c is the Compton frequency, then the equatorial orbital frequency is twice the Compton frequency, at all radii.

D. Motion in a radiation field

In the far radiation field of a single source, one has always that $S = \mathbf{E}\cdot\mathbf{B} = 0$ everywhere, so the derivation in section II C - which depends on the non-vanishing of the gradients of S - is invalidated. Going directly to the condition that the Lorentz force on the test particle vanish $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0}$, one immediately deduces that if $\mathbf{E} \cdot \mathbf{B} = 0$ everywhere, then

$$\mathbf{v} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \kappa\hat{\mathbf{B}} \tag{42}$$

where κ is a completely arbitrary function of space and time. Hence, in general, the determination of the motion of a massless charge is ill-posed if at any time $\mathbf{E} \cdot \mathbf{B} = 0$ everywhere. This finding applies to the case of motion in a radiation field irrespective of the relative contributions from the advanced and retarded fields. In practice of course, there is never a time where S vanishes everywhere, whether due to deviation from the idealized generator, or extrinsic noise. Such additional - second-order - contributions to the first-order orthogonal fields will have a first order effect on the velocity, and therefore no general statement about the solution is possible in these cases.

E. The two-body problem

All previous discussion of the motion of a source has been with the understanding that the fields acting on it are given. As pointed out in section II A, analysis based upon this assumption may be regarded as the first iteration in an infinite perturbative series. By contrast, in this section is briefly discussed a completely closed - non-perturbative - two-body interaction, equivalent therefore to having iterated the particle field interaction to convergence. Only very general properties of the interaction of two massless charges are discussed here. Explicit solutions of the two body problem will be presented elsewhere.

Let the electric field at $\mathbf{r}$ at current time t, due to a source at an earlier time t_{ret}, i.e., due to a source at $\mathbf{r}(t_{ret})$, be denoted by $\mathbf{E}_{ret} \equiv \mathbf{E}(\mathbf{r}, t|\mathbf{r}(t_{ret}))$, where t_{ret} is the solution of $t_{ret} = t - |\mathbf{r}(t) - \mathbf{r}(t_{ret})|$. With similar notation for the magnetic field, the relation between retarded $\mathbf{E}$ and $\mathbf{B}$ fields from a single source can be written ([18])

$$\mathbf{B}_{ret} = \hat{\mathbf{s}}_{ret} \times \mathbf{E}_{ret}; \qquad \hat{\mathbf{s}}_{ret} \equiv \frac{\mathbf{x} - \mathbf{x}(t_{ret})}{|\mathbf{x} - \mathbf{x}(t_{ret})|}. \tag{43}$$

It is deduced that the retarded fields of a single source give $\mathbf{B}_{ret}{\cdot}\mathbf{E}_{ret} = 0$ everywhere. As discussed above, in such circumstances, the problem is ill-posed and Eq.(20) is insufficient to determine the velocity of a test charge. Specifically, the component of the velocity of the test charge in the direction of the $\mathbf{B}$ field is undetermined. However, in our case retarded and advanced fields are mandatory, and the total (time-symmetric, direct action) fields are

$$\mathbf{E}(\mathbf{x}, t) = \frac{1}{2}(\mathbf{E}_{ret} + \mathbf{E}_{adv}), \quad \mathbf{B}(\mathbf{x}, t) = \frac{1}{2}(\mathbf{B}_{ret} + \mathbf{B}_{adv}). \tag{44}$$

Their scalar product is

$$\begin{aligned}\mathbf{B}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) &= \frac{1}{2}(\mathbf{E}_{ret} + \mathbf{E}_{adv}) \cdot (\hat{\mathbf{s}}_{ret} \times \mathbf{E}_{ret} + \hat{\mathbf{s}}_{adv} \times \mathbf{E}_{adv}) \\ &= \frac{1}{2}(\hat{\mathbf{s}}_{ret} - \hat{\mathbf{s}}_{adv}) \cdot (\mathbf{E}_{ret} \times \mathbf{E}_{adv})\end{aligned} \tag{45}$$

which is not zero in general, so the massless two-body problem is not ill-posed.

The 'no-interaction' theorem of Currie, Jordan, and Sudarshan [32] asserts that the charged particles can move only in straight lines if energy, momentum and angular momentum are to be conserved; the theorem effectively prohibits *any* EM interaction if a Hamiltonian form of the theory exists. Hill [22], Kerner [23], and others have observed that the prohibitive implication of the theorem can be circumvented if the canonical Hamiltonian coordinates are not identified with the physical coordinates of the particles. (Trump and Shieve [21] claim that the original proof is logically circular.) The theorem applies specifically to direct action CED written in terms of a single time variable. With superluminal trajectories, particle number is not generally conserved (in t-time) because some Lorentz frames will see time-reversals, leading one to doubt that this multi-time version of direct action EM can be cast into a single-time Hamiltonian form, putting it beyond the scope of the no-interaction theorem.

In section II D it was argued that the natural segmentation of an unconstrained trajectory is into sub-luminal and superluminal pieces. For as long as each source remains exclusively superluminal or exclusively sub-luminal, the situation remains a two-body problem, and the conclusions of this section apply. If however a trajectory crosses its own light cone (emanating from any 4-point) then that trajectory is effectively the trajectory of more than one particle, each segment generating its own advanced and retarded fields. In this case the above conclusions pertaining to the two-body will continue to apply only if there is just one trajectory, and that trajectory makes just one light-cone crossing in its entire history.

IV. DISCUSSION AND SPECULATION

A. Relationship to QM

It observed that the particle does not respond to force in the traditional sense of Newton's second law. Indeed, its motion is precisely that which causes it to feel no force, Eq.(10). Yet its motion is nonetheless uniquely prescribed by the (here misleadingly termed) 'force-fields' $\mathbf{E}$ and $\mathbf{B}$. These fields still decide the particle trajectory (given some initial condition), just as the Lorentz force determines the motion of a massive particle (again, given some initial condition). But the important difference between this and 'regular' classical electrodynamics is that here the local and instantaneous value of the external fields decide the *velocity* rather than the acceleration.

It is also observed that each term in the denominator and numerator of Eq.(20) is proportional to the same power (i.e. cubic) of the force-field components in $\mathbf{E}$ and $\mathbf{B}$. Hence, in the particular case of radiation fields wherein the magnitudes E and B are equal, the equation of motion of the massless test charge is insensitive to the fall-off of intensity from the radiating source.

These two qualities of the response to external fields - velocity rather than acceleration, and insensitivity to magnitude - are shared by the Bohm particle in the de Broglie-Bohm presentation of QM, ([33],[34],[35]) suggestive, perhaps, of a relation between the Bohm point and the massless classical charge.

A stumbling block appears to be that the Schrödinger and ('first quantized') Dirac wavefunctions are not fields in an (a priori) given space-time, in the manner of classical EM, to which *any* and all charges respond. Rather, the multi-particle Schrödinger and Dirac wavefunctions have as many spatial coordinate triples as there are particles (i.e., they exist in a direct product of 3-spaces). But in fact this characteristic is already a property of direct action without self action. To see this, note that for two bodies Eq.(6) becomes

$$F^{\nu\mu}_{(2)}\left(x_{(1)}\right)\frac{dx_{(1)}}{d\lambda}=0,\qquad F^{\nu\mu}_{(1)}\left(x_{(2)}\right)\frac{dx_{(2)}}{d\lambda}=0 \tag{46}$$

where $F^{\nu\mu}_{(2)}\left(x_{(1)}\right)$ is the field at $x_{(1)}(\lambda)$ due to the total future and historical contributions from the particle at $x_{(2)}(\kappa)$ such that $\left(x_{(1)}(\lambda)-x_{(2)}(\kappa)\right)^2=0$. The point is that if both particles pass just once through the 4-point ξ say, then, in general, the forces acting on each at that point are not the same: $F^{\nu\mu}_{(2)}(\xi)\neq F^{\nu\mu}_{(1)}(\xi)$. Thus, in common with QM, the fields can no longer be considered as existing in a given space-time to which any and all charges respond.

In the limit that the trajectory is sufficiently dense, it will no longer be possible to identify individual trajectories in any 4-volume, and one must go over to a continuum description. In that limit w defined in Eq.(15) becomes a Lorentz 4-vector field. The 4-divergence is

$$\partial_\mu w^\mu=-\left(\partial_\mu\widetilde{F}^{\mu\nu}\right)(\partial_\nu S)-\widetilde{F}^{\mu\nu}\partial_\mu\partial_\nu S=0. \tag{47}$$

(That the first term is zero follows from Maxwell's equation $\partial_\mu\widetilde{F}^{\mu\nu}=0$ provided there exists no magnetic current [31], and the second term is zero due to the anti-symmetry of $\widetilde{F}$.) Interpreting Eq.(47) as a continuity equation, $w_0=\mathbf{B}\cdot\nabla S$ is the density of a conserved charge, for which $\mathbf{w}=\mathbf{E}\times\nabla S-\mathbf{B}\partial S/\partial t$ is the current density. Of course, this is still a very long way from quantum field theory; w here is still a classical field.

B. Electron mass

Though these are encouraging signs, it cannot be claimed that the points raised above amount to confirmation of a correspondence between massless CED and QM. In any case, one cannot expect convergence between the two theories for as long as the intrinsically massless electron has not acquired mass (extrinsically); further development demands a convincing electromagnetic explanation of the origin of extrinsic mass - at least for the electron. There follows a sketch of an argument that it is hoped provides some justification for further investigation in that direction.

It is observed that in response to the given EM fields, the massless charge in question (here; the 'primary' source) generates 'out'-fields. These fields then add to the existing (first order given) fields so that at the next order, they must be taken into account when computing the motion of *other* ('secondary') sources that cross the past and future light cones of the primary source. I.E. they add to the 'in'-fields of those sources. Consequently the out-fields of the secondary sources are themselves to some degree affected by the out-fields of the primary source. So far all this applies without qualification to regular - time-asymmetric - EM. But here the difference is that all fields are alleged to be time-symmetric. With this novel amendment, it follows that the in-field arriving at the primary particle must contain some fraction that is a function of its own out-field. That is, some of the field arriving at the source must be a function of *the same field* leaving the primary source (at the present!) on a double light cone impacting distant sources at both past and future times. In the particular case that the response of these other sources (to this *incremental* field, say) is linear, then these same fields are reflected - effectively retransmitted - as both advanced and retarded waves, some of which must therefore return to the time of the original generation to arrive *in phase* with the primary source that emitted them.

It is important that this novel coherent combination of fields is distinguished from the self-field that dominates the traditional classical action and gives rise to infinite self energy. *That* field was excluded 'by fiat' in section I B. Instead, the fields discussed here arise only by virtue of reflection from other sources, and, unlike the traditional self-fields of CED, their energy is finite. In fact, under plausible assumptions which will be discussed elsewhere, these fields explain the Dirac large number coincidence, $m_e \simeq e^2\sqrt{N_H}/R_H$, as arising out of the self-consistency condition encumbent on symmetrically generated advanced and retarded fields. Broadly then, the speculation given here is that electron rest mass arises from EM field energy with direct self action and self energy excluded.

The system energy density is

$$\begin{aligned}\Theta_{00}\left(x\right) &= \frac{1}{2}\sum_{\substack{k,l=1\\k\neq l}}\left(\mathbf{E}_{(k)}\left(x\right)\cdot\mathbf{E}_{(l)}\left(x\right)+\mathbf{B}_{(k)}\left(x\right)\cdot\mathbf{B}_{(l)}\left(x\right)\right) \\ &= \frac{1}{2}\left(E^2\left(x\right)+B^2\left(x\right)\right)-\frac{1}{2}\sum_{l=1}\left(E^2_{(l)}\left(x\right)+B^2_{(l)}\left(x\right)\right)\end{aligned} \tag{48}$$

where each non-subscripted field is the traditional sum of contributions from all sources:

$$\mathbf{E}\left(x\right)=\sum_{l=1}\mathbf{E}_{(l)}\left(x\right), \qquad \mathbf{B}\left(x\right)=\sum_{l=1}\mathbf{B}_{(l)}\left(x\right). \tag{49}$$

With the self energy terms excluded, $\Theta_{00}\left(x\right)$ given in Eq.(48) is not positive definite. In fact, if the fields are uncorrelated, the sum is zero. In discussing Eq.(48) Leiter [29] is concerned to explain the predominance of exclusively retarded radiation in a theory in which electrons 'already' have mechanical mass. Consequently his fields are asymmetric combinations of advanced and retarded influences of a source. Here however, in discussing the bare massless condition at zero Kelvin, the fields in Eq.(48) are assumed to be perfectly time-symmetric. Then, if the self-consistency argument is accepted, one can say only that, on average, $\Theta_{00}(x)$ will be positive *close* (depending on the wavelengths of the EM fields) to the source.

The speculation then is that the 'observed' mass energy is this localized EM field energy. The idea extends the role of the distant, relatively cold absorbers employed by Wheeler and Feynman to explain the emergence of radiation in a time-symmetric theory; here, the distant charges continue to respond to EM fields at zero Kelvin - when there is no (exclusively retarded) radiation. That is, at all times they generate and reflect advanced and retarded EM fields, regardless of the temperature.

C. More on self action

In section I B the choice was made to deal with infinite self energy by excluding self action by fiat and adopting the direct action version of EM. The distribution of particle labels in Eqs.(3-5) enforce exclusion of 'diagonal' terms, which connote self-interaction. However, a finding of this investigation is that a massless particle in a given EM field obeying Eq.(25) can travel at both sub-luminal and superluminal speeds, which behavior undermines the labeling scheme. To see this, with reference to Fig. 4a, if the particle never achieves light speed, then clearly it will never cross any light cone emanating from any point on that trajectory. That is, the particle will never see its own light cone. Similarly for a particle that is always superluminal. But with reference to Fig. 4b, a trajectory with both sub-luminal and superluminal segments necessarily intersects its own light cone. If the whole trajectory is deemed to be non-self-interacting, in accordance with the fiat of no self action, then these points of electromagnetic contact cannot contribute to the action. Yet these points of interaction are similar in character to the 'genuine' - and therefore admitted - points of contact between any two different trajectories (if indeed there are multiple, distinguishable trajectories, each with their own starting points and end points). The problem is that the 'no self action' rule, necessary for masslessness of the bare charge, now impacts points of contact that are quite different to the infinitesimally local self action, i.e. $y = x$ in Eq.(4), that was the original target of the rule. In order that these distant points on the same trajectory conform to the fiat and be excluded from self action, it must be supposed that the trajectory, even after any number of time reversals, forever distinguish itself from other trajectories across all space-time, which requires that each trajectory carry a unique label (quite apart from its charge and state of motion). In addition to its intrinsic unattractiveness, this strategy is unappealing because it precludes the possibility all electrons and positrons can be described by just one trajectory.

In order to save the masslessness conjecture then, the alternative must be considered that electromagnetic contact is permitted between distant points on the *same* trajectory, whilst contact at infinitesimally local points is rendered finite (or zero). If such a solution exists, the result will be that the distinguishing labels will no longer be needed, and all electrons and positrons could then be regarded as segments on a single closed time-reversing trajectory. Then it will be possible to replace the action Eq.(29) with:

$$I = \frac{e^2}{8\pi} \int d\kappa \int d\lambda J \left(x\left(\kappa\right), x\left(\lambda\right), u\left(\kappa\right), u\left(\lambda\right)\right) \tag{50}$$

for some function J. If the light cone direct action form is retained, then

$$I = \frac{e^2}{8\pi} \int d\kappa \int d\lambda \delta \left(\left(x\left(\kappa\right) - x\left(\lambda\right)\right)^2\right) K \left(u\left(\kappa\right), u\left(\lambda\right)\right) \tag{51}$$

for some function K. If this form is assumed, the argument above amounts to the requirement that the contribution to the action from the zero of the delta function argument at $\kappa = \lambda$ is finite.

One possibility is simply to modify the action so that K is zero at this point. For example, it is readily seen that

$$K = \frac{1}{2} \left(u\left(\kappa\right) - u\left(\lambda\right)\right)^2 = \frac{1}{2} \left(u^2\left(\kappa\right) + u^2\left(\lambda\right)\right) - u^\mu\left(\lambda\right) u_\mu\left(\kappa\right) \tag{52}$$

achieves the stated end. It interprets the product of the two 4-velocities as the cross terms in a perfect square, the latter obviously vanishing at $\kappa = \lambda$. A consequence is that the action Eq.(29) is augmented by

$$I_{new} = \frac{e^2}{8\pi} \int d\kappa \int d\lambda u^2\left(\kappa\right) \delta \left(\left(x\left(\kappa\right) - x\left(\lambda\right)\right)^2\right), \tag{53}$$

and the task then is to demonstrate the plausibility of this new contribution. A nice feature of this modification is that it automatically implements the Wheeler-Feynman fiat of no (instantaneous) self action, whilst leaving intact action at a distance, which is now interpreted as action between different *segments* of the same trajectory.

Eq.(52) is offered only as an illustrative example of how the stated goal might be implemented through modification of the action. Another method has been reported elsewhere ([30]). But other interesting possibilities exist, a report on which is in progress.

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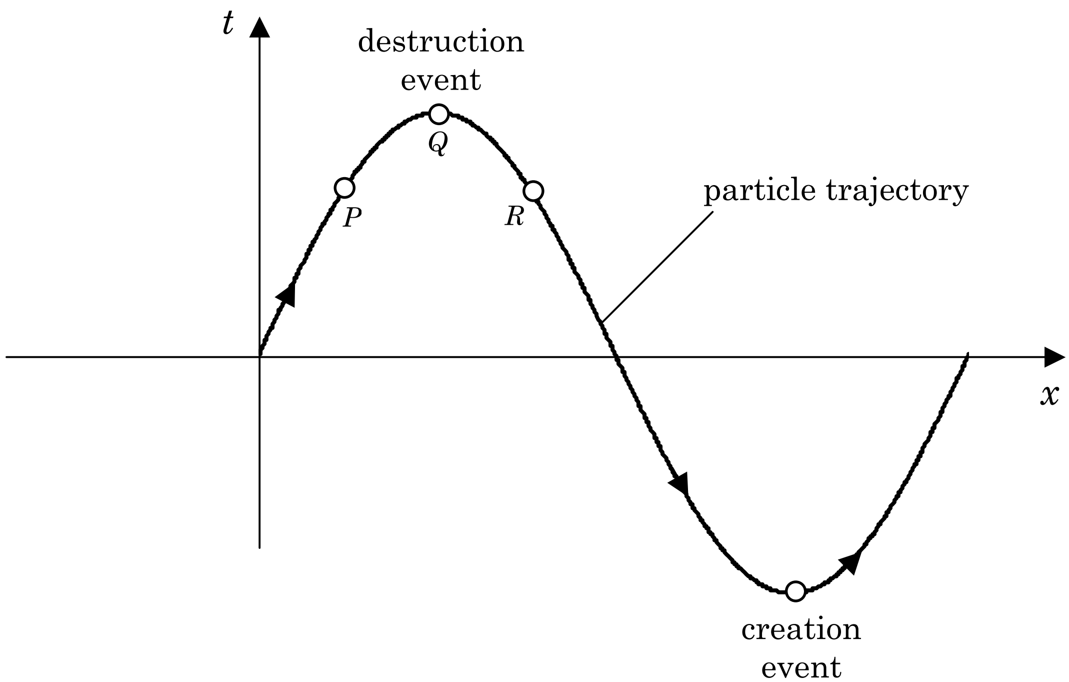

Figure 1

A trajectory that reverses in ordinary time may be interpreted as giving rise to pair creation and pair destruction events.

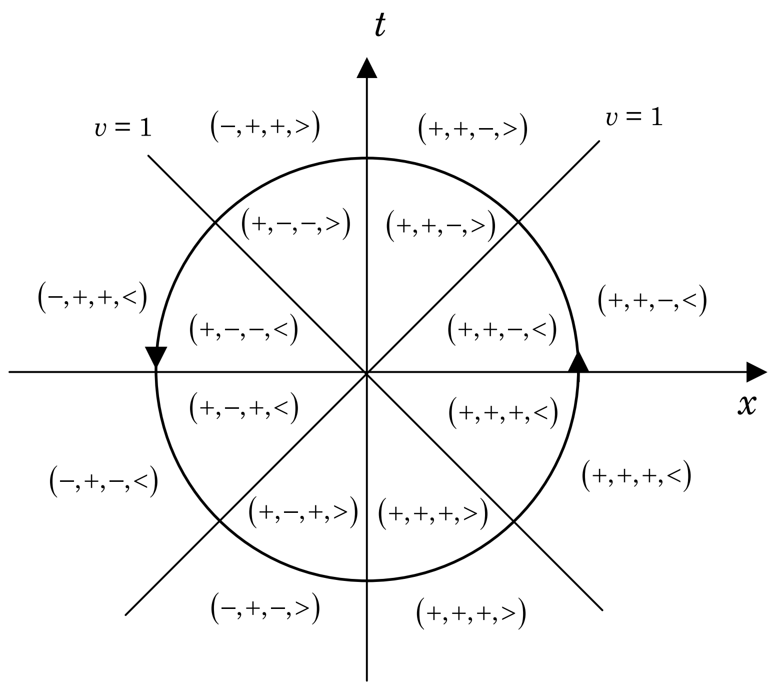

Figure 2

The bracketed symbols denote (sign of charge, sign of $dt/d\tau$, sign of $dx/d\tau$, speed: > or < speed of light). In the interior of the circle the sign of the charge is fixed, but can take either value in the absence of any other context- the choice that it is positive is arbitrary. In the exterior of the circle, the bracketed symbols denote the CPT-invariant alternative designation in which $dt/d\tau$ is always positive.

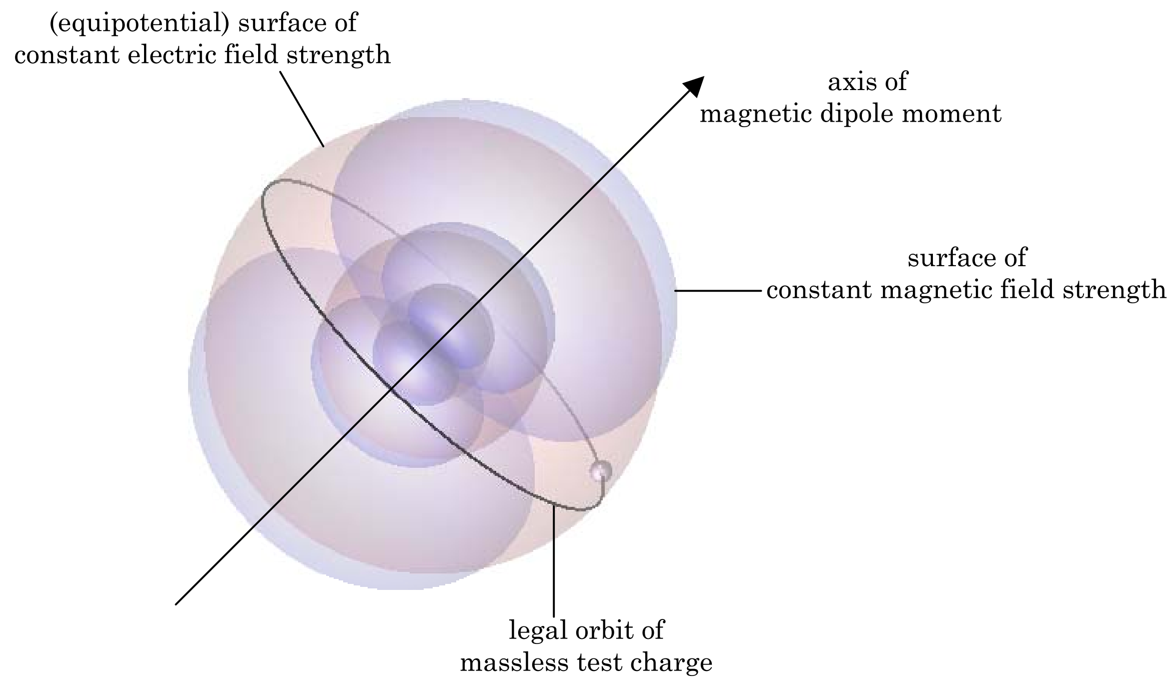

Figure 3

Orbit of massless charge in a field due to a single electric charge with a magnetic dipole.

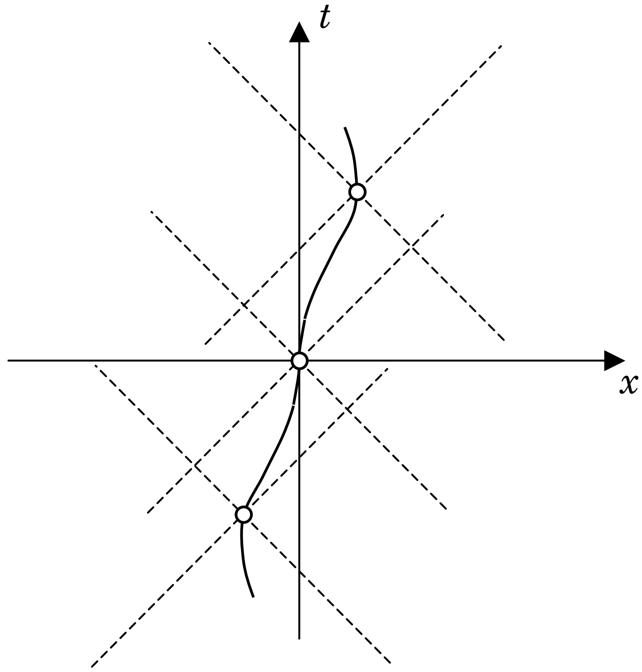

Figure 4 (a)

Sub-luminal trajectory showing light cones from three selected space-time points.

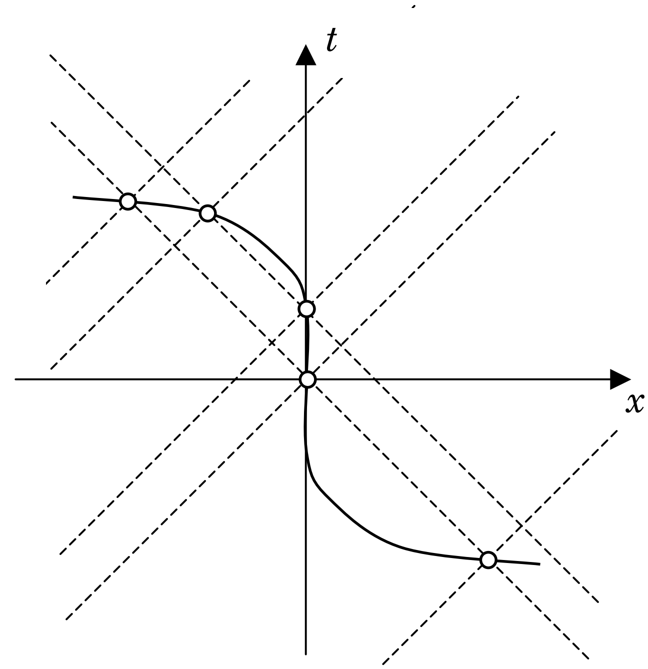

Figure 4 (b)

Presence of both sub-luminal and superluminal speeds necessarily gives rise to self-interaction, shown here by dashed lines connecting selected space-time points (circles) on the trajectory.