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Darwin-Lagrangian analysis for the interaction of a point charge and a magnet: considerations related to the controversy regarding the Aharonov-Bohm and Aharonov-Casher phase shifts. (English summary)
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This paper challenges the assumption that Lorentz forces can be neglected in experimental tests of the Aharonov-Bohm effect and therefore, by implication, challenges the validity of the proof of the local reality of the vector potential. The major part of the analysis is of a single initially static classical point (test) charge q at $\mathbf{r}_q$ in the vicinity of a classical hydrogen-like atom that serves as the source of a single magnetic dipole μ. The focus is primarily on the case that the test charge is in the same plane as the atom (normal to μ). Equations of motion for the point charge, the centre of mass of the atom, and the electric dipole of the atom are derived from the Darwin Lagrangian, correct to order $1/c^2$. The author shows how the orbit of the hydrogen electron is destabilized by the presence of the test charge, resulting in (i) a secular motion of the center of magnetic field energy, and (ii) a secular electric field which then acts back on the test charge at order $1/c^2$. The sign of the resulting force is independent of the signs of charges in the atom. It is demonstrated that the result conserves energy and momentum.

The author points out that this result is not accommodated by analyses that assume a priori that the charges and currents in the magnet are unaffected by the presence of a charge. Those analyses assume that the vector product of the electric field of the test charge with the magnetic moment μ represents a hidden momentum in magnets, rather than—as here—a motion of the center of energy of the magnetic field $\mathbf{B}_\mu$ due to μ.

The result serves to demonstrate the presence of a perhaps unexpected Lorentz back-reaction on the test charge, but is not directly applicable to practical AB configurations in which the solenoid is shielded from electric fields. In a semi-quantitative discussion of that case the author points out that there remains a back-reaction on the test charge at order $1/c^2$ when it is in motion. The resulting magnetic field exerts a Lorentz force on the (moving) charges in the magnetic dipole atom which, at the next iteration, cause a change in the magnetic field experience by the test charge. That is, the charge at $\mathbf{r}_q$ experiences a force due to the dependence of the contribution to the (total) magnetic field energy from $\int dV \mathbf{B}_\mu \cdot \mathbf{B}_q$ on $\mathbf{r}_q$, this despite the fact that $\mathbf{B}_\mu(\mathbf{r}_q) = \mathbf{0}$. It is claimed that experimental tests of the AB effect to date do not (effectively) shield magnetic fields of the test charge from the solenoid, and are therefore vulnerable to a classical Lorentz-force interpretation.

Reviewed by *Michael Ibison*