ZERO-POINT FLUCTUATIONS OF THE VACUUM AS THE SOURCE OF ATOMIC STABILITY AND THE GRAVITATIONAL INTERACTION

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INTRODUCTION

In modern theory the vacuum is more properly characterized as a plenum than as a void. This is due to the fact that, even in the absence of matter, the vacuum is the seat of zero-point-fluctuation (ZPF) energy densities of such fields as the vacuum electromagnetic field, which is the focal point of our study here. The energy density associated with this (usually unobserved) background is, formally, considered to be infinite; with appropriate high-frequency cutoffs the ZPF energy density is still conservatively estimated to be on the order of nuclear energy densities or greater.

The enormity of the figures describing the vacuum electromagnetic zero-point energy has led theorists to question from time to time whether these numbers should be taken seriously, or whether they are due to some defect or misinterpretation of the theory. However, over the years, even though this energy density has not been measured directly, certain physical consequences due to its presence have been measured. One very physical example is the unique zero-point quantum force between closely spaced metal plates known as the Casimir force (Casimir (1), Fierz (2), Marshall (3), Boyer (4)). The Casimir force results from the redistribution of viable normal modes (and hence in the associated vacuum electromagnetic ZPF energy) as the distance between the plates changes. As a result of this and other similar prediction and measurement, the reality of the zero-point energy is now accepted as part of the lexicon of modern quantum theory, although there is still discussion as to whether it ought to be considered as "real" or "virtual" (Boyer et al (5)). Of particular significance with regard to its lack of direct observability is the fact that its spectral distribution,

$$\rho(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3} \qquad (1)$$

is Lorentz invariant, which derives specfically from the spectrum's cubic dependence on frequency. The cubic spectrum is unique in its property that delicate cancellation of Doppler shifts with velocity boosts leaves the spectrum Lorentz-invariant.

Now yet further demonstration of the reality and significance of this ubiquitous energy density has turned up in two studies by the author, the first dealing with the microcosm of the atom (Puthoff (6)), the second with the macrocosm of gravitational interaction (Puthoff (7)). Specifically, the first addresses and resolves the issue of radiative collapse of the Bohr atom, while the second pertains to the development of a model, originally proposed by Sakharov, of gravity not as a separately-existing fundamental force, but rather as an induced effect associated with the zero-point fluctuations of the vacuum, along the lines of the van der Waals and Casimir forces.

GROUND STATE OF HYDROGEN AS A ZPF-DETERMINED STATE

We begin with a discussion of the Bohr model of the atom. When it was first recognized that the atom could be likened to a small solar system in which electron planets orbited a nuclear sun, theorists were puzzled as to why the electrons in their tightly-curved orbits did not radiate their energy away and spiral into the nucleus as predicted by classical theory. At the time the best answer was that it appeared to be a property of special quantum states, and no further elaboration was possible. Now, however, this issue has been re-examined, this time taking into account what has been learned in the interim about the effects of zero-point energy. It is shown that the electron in the ground state (lowest-energy state) of the hydrogen atom indeed can be seen as continually radiating its energy away as predicted by classical theory, *but* also absorbing energy from the ever-present sea of electromagnetic zero-point energy in which the atom is immersed, and an hypothesized equilibrium between these two processes leads to the correct values for the parameters known to define the ground-state orbit.

To show this, we consider a conceptually-simple, classical model (but including ZPF) in which the Bohr-atom electron interacts with a background of random classical electromagnetic ZPF radiation with energy spectrum given by (1). This treatment of quantum field-particle interactions on the basis of a classical ZPF background constitutes an analysis technique known in the literature as stochastic electrodynamics (SED) (Boyer et al (8)). SED is a well-defined framework that has a long history of success in yielding precise quantitative agreement with full QED treatments of such topics as the Planck blackbody radiation spectrum (Boyer (9)), Casimir ((3), (4), Lifshitz (10)) and van der Waals forces (Boyer (11)), and the thermal effects of acceleration through the vacuum (Boyer (12)), all originally thought to be soluble only within the quantum formalism.

In the SED approach the vacuum is assumed to be filled with random classical zero-point electromagnetic radiation whose Fourier composition underlies the spectrum given in (1). Written as a sum over plane waves, the random radiation, which is homogeneous, isotropic and Lorentz invariant, can be expressed as

$$\underline{E}^{zp}(\underline{r},t) = \mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k\,\hat{\underline{\epsilon}}\left(\frac{\hbar\omega}{8\pi^3\epsilon_0}\right)^{1/2} e^{i\underline{k}\cdot\underline{r}-i\omega t+i\theta(\underline{k},\sigma)}, \qquad (2)$$

$$\underline{H}^{zp}(\underline{r},t) = \mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k\,(\hat{\underline{k}}\times\hat{\underline{\epsilon}})\left(\frac{\hbar\omega}{8\pi^3\mu_0}\right)^{1/2} e^{i\underline{k}\cdot\underline{r}-i\omega t+i\theta(\underline{k},\sigma)}, \qquad (3)$$

where $\sigma = 1,2$ denote orthogonal polarizations, $\hat{\epsilon}$ and $\hat{k}$ are orthogonal unit vectors in the direction of the electric field polarization and wave propagation vectors, respectively, $\theta(\underset{\sim}{k},\sigma)$ are random phases distributed uniformly on the interval $0 - 2\pi$ (independently distributed for each $\underset{\sim}{k},\sigma$), and $\omega = kc$.

We begin our discussion of the ground-state Bohr orbit by considering a one-dimensional charged harmonic oscillator of natural frequency ω_o, located at the origin and immersed in zero-point radiation. For orientation along the x axis, the (nonrelativistic) equation of motion for a particle of mass m and charge e, including radiation damping, is given by

$$m\ddot{x} + m\omega_o^2 x = \left(\frac{e^2}{6\pi\epsilon_o c^3}\right)\dddot{x} + eE_x^{zp}(0,t) \quad . \tag{4}$$

Substitution of (2) into (4) leads to the following expression for the velocity, which is of interest:

$$v = \dot{x} = \frac{e}{m}\,\mathrm{Re}\sum_{\sigma=1}^{2}\int d^3k\,(\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})\left(\frac{\hbar\omega}{8\pi^3\epsilon_o}\right)^{1/2}\left(\frac{-i\omega}{D}\right) \tag{5}$$

$$\times\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t + i\theta(\underset{\sim}{k},\sigma)} \quad ,$$

where

$$D = -\omega^2 + \omega_o^2 - i\Gamma\omega^3 \quad , \tag{6}$$

$$\Gamma = \frac{e^2}{6\pi\epsilon_o mc^3} \quad . \tag{7}$$

From (2) and (5) we now calculate the average power absorbed from the zero-point background as

$$\langle P^{ABS}\rangle = \langle \underset{\sim}{E}\cdot\underset{\sim}{v}\rangle = \langle e\,\underset{\sim}{E}^{zp}\cdot\underset{\sim}{v}\rangle$$

$$= \frac{1}{2}\mathrm{Re}\left\langle\frac{e^2}{m}\sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k\int d^3k'\,(\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})(\hat{\underset{\sim}{\epsilon}}'\cdot\hat{\underset{\sim}{x}})\left(\frac{\hbar\omega}{8\pi^3\epsilon_o}\right)^{1/2}\left(\frac{\hbar\omega'}{8\pi^3\epsilon_o}\right)^{1/2}\left(\frac{i\omega'}{D^*}\right)\right.$$

$$\left.\times\exp[i(\underset{\sim}{k}-\underset{\sim}{k}')\cdot\underset{\sim}{r} - i(\omega-\omega')t + i\theta(\underset{\sim}{k},\sigma) - i\theta(\underset{\sim}{k}',\sigma')]\right\rangle \quad , \tag{8}$$

where use of the complex conjugate and the notation (1/2)Re stems from the use of exponential notation. Eq. (8) can, however, be simplified to

$$\langle P^{ABS}\rangle = \frac{1}{2}\mathrm{Re}\left[\frac{e^2}{m}\sum_{\sigma=1}^{2}\int d^3k\,(\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})^2\,\frac{\hbar\omega}{8\pi^3\epsilon_o}\,\frac{i\omega}{D^*}\right] \quad , \tag{9}$$

where averaging over random phases involves the use of

$$\langle \exp[i(\underline{k}-\underline{k}')\cdot\underline{r}-i(\omega-\omega')t+i\theta(\underline{k},\sigma)-i\theta(\underline{k}',\sigma')]\rangle=\delta_{\sigma\sigma'}\delta_{\omega\omega'}\delta^3(\underline{k}-\underline{k}') . \tag{10}$$

With $\int d^3k \rightarrow \int d\Omega_k \int dk\, k^2$ (9) can be rewritten as

$$\langle P^{ABS}\rangle=\frac{1}{2}\mathrm{Re}\left[\frac{e^2}{m}\int d\Omega_k\left(\sum_{\sigma=1}^{2}(\hat{\underline{\epsilon}}\cdot\hat{\underline{x}})^2\right)\int dk\, k^2\ \frac{\hbar\omega}{8\pi^3\epsilon_0}\ \frac{i\omega}{D^*}\right] . \tag{11}$$

We further note that, with the sum over polarizations given by

$$\sum_{\sigma=1}^{2}[\hat{\underline{\epsilon}}(\underline{k},\sigma)\cdot\hat{\underline{x}}_i][\hat{\underline{\epsilon}}(\underline{k},\sigma)\cdot\hat{\underline{x}}_j]=\delta_{ij}-(\hat{\underline{k}}\cdot\hat{\underline{x}}_i)(\hat{\underline{k}}\cdot\hat{\underline{x}}_j) , \tag{12}$$

the angular integration in k takes the form

$$\int d\Omega_k\left(\sum_{\sigma=1}^{2}(\hat{\underline{\epsilon}}\cdot\hat{\underline{x}})^2\right)=\int d\Omega_k[1-(\hat{\underline{k}}\cdot\hat{\underline{x}})^2]=\frac{8}{3}\pi . \tag{13}$$

Substitution of (13) into (11), and a change of variables to ω = kc, then leads to

$$\langle P^{ABS}\rangle=\frac{e^2\hbar}{6\pi^2\epsilon_0 mc^3}\mathrm{Re}\left(\int_0^\infty\frac{i\omega^4\, d\omega}{-\omega^2+\omega_0^2+i\Gamma\omega^3}\right)$$

$$=\frac{e^2\hbar}{6\pi^2\epsilon_0 mc^3}\int_0^\infty\frac{\Gamma\omega^7\, d\omega}{(\omega_0^2-\omega^2)^2+\Gamma^2\omega^6} . \tag{14}$$

Because of the smallness of Γ for a particle with the charge-to-mass ratio of an electron, for this case the integrand in (14) is sharply peaked around $\omega = \omega_0$. We can therefore invoke the standard resonance approximation, extending the limits of integration and replacing ω by ω_0 in all but the difference term. This yields

$$\langle P^{ABS}\rangle=\frac{e^2\hbar\omega_0^3}{12\pi\epsilon_0 mc^3}\int_{-\infty}^{\infty}\frac{1}{\pi}\ \frac{(\Gamma\omega_0^2/2)\, d\omega}{(\omega_0-\omega)^2+(\Gamma\omega_0^2/2)^2}$$

$$=\frac{e^2\hbar\omega_0^3}{12\pi\epsilon_0 mc^3} , \tag{15}$$

since the (Lorentzian lineshape) integral integrates to unity. In this result we thus have the final expression for the absorption of power from the random background zero-point field by a one-dimensional charged harmonic oscillator.

We now recognize that the Bohr-theory ground-state circular orbit of radius r_0 constitutes a pair of one-dimensional harmonic oscillators in a plane, oscillating in quadrature,

$$
\begin{aligned}
x &= x_0 \cos \omega_0 t \quad , \\
y &= y_0 \sin \omega_0 t = x_0 \sin \omega_0 t \quad , \\
r_0 &= (x^2 + y^2)^{1/2} = x_0 \quad .
\end{aligned}
\tag{16}
$$

Therefore, the power absorbed from the background by the electron in circular orbit is double that of (15), or

$$
\langle P^{ABS} \rangle_{CIRC} = \frac{e^2 \hbar \omega_0^3}{6\pi \epsilon_0 m c^3} \quad . \tag{17}
$$

The power radiated by the electron in circular orbit with acceleration A is given by the standard expression (Feynman et al (13))

$$
\langle P^{RAD} \rangle_{CIRC} = \frac{e^2 A^2}{6\pi \epsilon_0 c^3} = \frac{e^2 (r_0 \omega_0^2)^2}{6\pi \epsilon_0 c^3} = \frac{e^2 r_0^2 \omega_0^4}{6\pi \epsilon_0 c^3} \quad . \tag{18}
$$

Under the hypothesis that the ZPF-determined ground-state orbit is set by a balance between radiation emitted due to acceleration of the electron, and radiation absorbed from the zero-point background, we equate (17) and (18) to obtain

$$
m \omega_0 r_0^2 = \hbar \, . \tag{19}
$$

We have therefore, within the SED framework, and at the level of Bohr theory, obtained the desired result for the ground state of the hydrogen atom. Thus the ground-state orbit can be interpreted as set by a dynamic equilibrium in which collapse of the state is prevented by the presence of the zero-point energy. The significance of this observation is that the very stability of matter itself depends upon the underlying sea of electromagnetic zero-point energy density universally present throughout space.

GRAVITY AS A ZPF FORCE OF THE LONG-RANGE VAN DER WAALS TYPE

With regard to gravity, although much is known about its mathematical structure and its empirical effects, its fundamental nature is still not well understood. Whether addressed simply in terms of Newton's Law, or with the full rigor of general relativity, gravitational theory is basically descriptive in nature, without revealing the underlying dynamics for that description. As a result, attempts to unify gravity with the other forces (electromagnetic, strong and weak nuclear forces), or to develop a quantum theory of gravity, have foundered again and again on difficulties that can be traced back to a lack of understanding at a fundamental level. To rectify these difficulties, theorists by and large have resorted to ever-increasing levels of mathematical sophistication and abstraction, as in the recent development of supergravity and superstring theories.

Taking a completely different tack when addressing these difficulties in the sixties, the well-known Russian physicist Andrei Sakharov put forward the hypothesis that gravitation might not be a fundamental interaction at all, but rather a secondary or residual effect associated with other (non-gravitational) fields (Sakharov (14), Misner et al (15)). Specifically, Sakharov suggested that gravity could be understood as an induced effect brought about by changes in the quantum fluctuation energy (zero-point energy) of the vacuum due to the presence of matter. In this view the attractive gravitational force is more akin to the induced van der Waals and Casimir forces, than to the fundamental coulomb force. Although speculative when first introduced by Sakharov in 1967, this hypothesis has led to an ongoing literature on gravity as a symmetry-breaking effect in quantum field theory which continues to be of interest (Adler (16)).

This approach to gravity is addressed here in particularly concise form, with positive results. We show that gravitational mass and its associated gravitational effects emerge in a natural way from electromagnetic ZPF-induced particle motion. In brief, the gravitational interaction begins with the fact that a particle situated in the sea of electromagnetic ZPF develops a "jitter" motion (*zitterbewegung*). When two (or more) particles are in proximity, they are each influenced not only by the fluctuating background field, but also by the fields generated by the other particle(s), similarly undergoing *zitterbewegung* motion. The field-correlated motions of such particles then result in an inter-particle coupling that leads straightforwardly to the attractive gravitational force, with no free parameters to be determined. Gravity can thus be understood as a fluctuational force of the van der Waals type, although of much longer range than typical van der Waals forces because of involving the radiation rather than the (usual) induction fields.

To arrive at these results, basically we simply assemble together in a straightforward fashion previously-published results regarding ZPF models of van der Waals and related effects. When this is done, one finds the leading term in the interaction potential, previously unexamined, to be Newton's Law with no free parameters to be fixed. Because of its electromagnetic underpinning, gravitational theory in this form constitutes an "already-unified" theory.

We begin our exploration of the Sakharov viewpoint on the basis of a harmonic-oscillator model of the type used in the previous section. In its application, we represent matter as a collection of bound, charged point-mass particles (partons), in accordance with standard theory. In the development that follows it is not necessary to invoke the details of particular parton representations (e.g., families of fractionally-charged quarks) beyond certain general concepts, such as the "asymptotic freedom" of partons to respond to the high frequency components of the ZPF spectrum as essentially free particles.

The harmonic oscillator Eq. (4) can be written in the form

$$\ddot{\mathbf{p}} + \omega_0^2 \mathbf{p} = \Gamma \dddot{\mathbf{p}} + 6\pi\epsilon_0 c^3 \Gamma \mathbf{E}^{zp} \quad , \tag{20}$$

where we have introduced the dipole moment, $\mathbf{p} = q\mathbf{r}$, and the damping constant, $\Gamma = q^2/6\pi\epsilon_0 m_0 c^3$.

For the parton - ZPF interaction of interest, we treat the parton as a two-dimensional (rather than three-dimensional) oscillator, drawing on previous studies that model spin as the "internal" angular momentum asociated with two-dimensional fluctuation motion (Huang (17)). Also, because we are interested primarily in the particle's high-frequency fluctuation response to the ZPF, whose spectral density increases as ω^3, we neglect the binding-force term involving ω_0 (asymptotic freedom as it relates to the ZPF). Finally, we also neglect the radiation-damping force in comparison to the inertial force and ZPF driving terms.

Under the asymptotically-free-particle assumptions stated in the paragraph above, the x component of (20) takes the form (with $\hat{\mathbf{x}}$ a unit vector in the x direction)

$$\frac{d\dot{p}_x}{dt} = 6\pi\epsilon_0 c^3 \Gamma E_x^{zp} = 6\pi\epsilon_0 c^3 \Gamma \,\mathrm{Re}\left\{ \sum_{\sigma=1}^{2} \int d^3k \,(\hat{\boldsymbol{\epsilon}}\cdot\hat{\mathbf{x}}) \left(\frac{\hbar\omega}{8\pi^3\epsilon_0}\right)^{1/2} \times \exp\left[i\mathbf{k}\cdot\mathbf{r} - i\omega t + i\theta(\mathbf{k},\sigma)\right] \right\} \quad . \tag{21}$$

To obtain the expectation value of kinetic energy of the fluctuation motion, $\langle \mathcal{E} \rangle = \frac{1}{2} m_0 \langle \dot{r}^2 \rangle /2 \ = \langle \dot{p}^2 \rangle / 12\pi\epsilon_0 c^3 \Gamma$, we apply an integration and averaging procedure due to Rueda (18). The result is that $\langle \dot{p}_x^2 \rangle$ reaches

$$\langle \dot{p}_x^2 \rangle = 6\epsilon_0 \hbar c^3 \Gamma^2 \omega_c^2 \quad , \tag{22}$$

where ω_c is the assumed cutoff frequency, to be determined later. For the two-dimensional fluctuation motion assumed, $\langle \dot{p}^2 \rangle = 2\langle \dot{p}_x^2 \rangle$, which yields for the kinetic energy

$$\langle \mathcal{E} \rangle = \frac{\Gamma \hbar \omega_c^2}{\pi} \quad . \tag{23}$$

It is thus seen that the expectation value of the kinetic energy of parton fluctuation motion reaches a finite magnitude, limited by the finite value of the (as-yet-undetermined) ZPF cutoff frequency (Heitler (19)). Since the energy associated with this fluctuation motion is an "internal" particle energy, that is, not directly observable, we identify this energy as that corresponding to the rest-mass energy of the particle, m,

$$m = \frac{\langle \mathcal{E} \rangle}{c^2} = \frac{\Gamma \hbar \omega_c^2}{\pi c^2} \quad . \tag{24}$$

In this view the particle mass m is of dynamical origin, originating in parton-motion response to the electromagnetic zero-point fluctuations of the vacuum (footnote (20)). It is therefore simply a special case of the general proposition that the internal kinetic energy of a system contributes to the effective mass of that system (Bohm (21)). *As will be shown, it is this mass that is involved in the gravitational interaction.*

Let us now turn our attention to the interaction between particles. When two (or more) are in proximity, they are each influenced not only by the fluctuating background field, but also by the fields generated by the other particle(s), all similarly undergoing fluctuation motion. For the case of interest here (binding and radiation-damping forces neglected) Boyer has derived an expression for the interaction potential U for the retarded van der Waals forces at all distances between a pair of like particles, interacting with the classical background ZPF as assumed here. It is (Boyer (22))

$$U = -\frac{9}{4}\frac{\hbar c^3 \Gamma^2}{\pi} \mathrm{Re}\int_0^{u_c} du \frac{e^{-2uR}}{R^2}\left[1 + \frac{2}{uR} + \frac{5}{(uR)^2} + \frac{6}{(uR)^3} + \frac{3}{(uR)^4}\right] , \tag{25}$$

where $u = -i\omega/c$ and R is the distance between particles. The only difference here as compared to the derivation in Ref. 22 is the use of a finite cutoff frequency. (For those who might be more familiar with standard quantum calculations, this result has also been obtained by Renne (Renne (23)) and by Casimir and Polder (Casimir et al (24)) in QED calculations.)

We are now ready to apply this standard result to the gravitational problem. First, on the scale of interest in gravitation (distances large compared with the wavelengths of the predominant fluctuation frequencies) we need retain only the radiation-field contribution to the interaction potential, which is the first (unity) term in brackets. This differentiates the results to be derived here from the usual van der Waals effects involving the induction fields (remaining terms in brackets). Second, for the two-dimensional fluctuation motion assumed in our case (N = 2), geometrical considerations require that the above expression for U, derived for the case in which three degrees of freedom for particle motion were assumed, be reduced by a factor $(N/3)^2 = 4/9$ (Puthoff (7)). With these factors taken into account the solution to (25) becomes

$$U = -\frac{\chi}{\mathcal{R}}\left(\frac{\sin \mathcal{R}}{\mathcal{R}}\right)^2 , \tag{26}$$

where $X = \hbar\Gamma^2\omega_c^3/\pi$ and $\mathcal{R} = \omega_c R/c$. With the potential thus defined, the force is obtained from $\mathbf{F} = -\partial U/\partial R$.

We see therefore that the potential has the desired 1/R dependence required for gravity, modulated by a fine-structure overlay of the form $(\sin\mathcal{R}/\mathcal{R})^2$ which has a spatial periodicity characteristic of the cutoff (Planck) frequency ($\sim 10^{-33}$ cm). If we extract the leading (non-oscillatory) term, we find for the potential and force

$$U = -\frac{\hbar c \Gamma^2 \omega_c^2}{\pi R} + \cdots \quad , \tag{27}$$

$$F = -\frac{\hbar c \Gamma^2 \omega_c^2}{\pi R^2} + \cdots \quad . \tag{28}$$

A careful examination of the details of averaging over the rapid spatial variation shows that the particle experiences an average force $\langle F \rangle$ given by the leading term in (28) (see Appendix). With Γ in terms of m given by (24), $\langle F \rangle$ can be written in the form

$$\langle F \rangle = -\left(\frac{\pi c^5}{\hbar \omega_c^2}\right) \frac{m^2}{R^2} \quad . \tag{29}$$

At this point we note that the assumption of a Planck-like value for the cutoff frequency, $\omega_c = \sqrt{\pi c^5/\hbar G}$, would directly yield Newton's Law. However, we can obtain this value for the cutoff from fundamental principles, without assumption. We begin with the observation that in an accelerated frame the spectral distribution (1) takes the form (Boyer (25))

$$\rho(\omega)d\omega = \left(\frac{\omega^2}{\pi^2 c^3}\right)\left[1 + \left(\frac{a}{\omega c}\right)^2\right]\left[\frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\frac{2\pi c\omega}{a}} - 1}\right]d\omega \quad , \tag{30}$$

where a is the proper acceleration relative to a Lorentz frame.

Of special interest here is not the Planck-like (exponential) term which is of interest with regard to thermal effects of acceleration through the vacuum, but rather the leading terms

$$\rho'(\omega) = \rho_0(\omega) + \Delta\rho'(\omega) = \frac{\hbar\omega^3}{2\pi^2 c^3} + \frac{\hbar\omega a^2}{2\pi^2 c^5} \quad . \tag{31}$$

These indicate that an accelerated observer would see the background ZPF spectrum augmented by a term proportional to the square of the acceleration. Application of the principle of equivalence then indicates that the additional spectral contribution seen in a frame with acceleration **a** should also be seen in a nonaccelerated frame with local gravitational field **g** produced by a mass m. Setting **g** = - **a** = - Gm/r^2, we obtain

$$\Delta\rho'(\omega) = \frac{\hbar\omega}{2\pi^2 c^3}\ \frac{G^2 m^2}{r^4} \quad . \tag{32}$$

We turn our attention now to the fields generated by the ZPF-induced fluctuation motion to determine whether the above additional contribution predicted by the equivalence principle is in fact generated. Considering, say, the x component of motion, we find that an assumed $e^{-i\omega t}$ time dependence substituted into (20) yields for the magnitude of any particular frequency component

$$\tilde{p}_x(\omega) = -\frac{6\pi\epsilon_0 c^3\Gamma}{\omega^2}(\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})\,\tilde{E}^{zp}(\omega) \quad , \tag{33}$$

where the over-tilde designates the magnitude of a frequency component, and once again we have neglected the binding and radiation-damping forces. This expression can then be combined with the ZPF-field expressions (2) and (3), and the standard oscillating dipole formulae (Stratton (26))

$$\underset{\sim}{E}_d(\omega) = \mathrm{Re}\left\{\frac{1}{4\pi\epsilon_0}\tilde{p}\,e^{-i\omega t}\underset{\sim}{G}\right\} \quad , \tag{34}$$

$$\underset{\sim}{H}_d(\omega) = \mathrm{Re}\left\{\frac{c}{4\pi}\tilde{p}\,e^{-i\omega t}\underset{\sim}{F}\right\} \quad , \tag{35}$$

to yield expressions for the dipole fields generated by the fluctuation motion; viz,

$$\underset{\sim}{E}_d = -\mathrm{Re}\left\{\frac{1}{4\pi\epsilon_0}\sum_{\sigma=1}^{2}\int d^3k\left(\frac{6\pi\epsilon_0 c^3\Gamma}{\omega^2}\right)\left(\frac{\hbar\omega}{8\pi^3\epsilon_0}\right)^{1/2}(\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})\,e^{-i\omega t+i\theta(\underset{\sim}{k},\sigma)}\underset{\sim}{G}\right\} , \tag{36}$$

$$\underset{\sim}{H}_d = -\mathrm{Re}\left\{\frac{c}{4\pi}\sum_{\sigma=1}^{2}\int d^3k\left(\frac{6\pi\epsilon_0 c^3\Gamma}{\omega^2}\right)\left(\frac{\hbar\omega}{8\pi^3\epsilon_0}\right)^{1/2}(\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})\,e^{-i\omega t+i\theta(\underset{\sim}{k},\sigma)}\underset{\sim}{F}\right\} , \tag{37}$$

where, with $\hat{\mathbf{r}}$ a unit vector in the direction joining the dipole to the field evaluation point,

$$\underset{\sim}{G} = k^3 e^{ikr}\left\{(\hat{\underset{\sim}{r}}\times\hat{\underset{\sim}{x}})\times\hat{\underset{\sim}{r}}\left(\frac{1}{kr}\right)+\left[3\hat{\underset{\sim}{r}}(\hat{\underset{\sim}{r}}\cdot\hat{\underset{\sim}{x}})-\hat{\underset{\sim}{x}}\right]\left[\frac{1}{(kr)^3}-\frac{i}{(kr)^2}\right]\right\} , \tag{38}$$

$$\underset{\sim}{F} = k^3 e^{ikr}(\hat{\underset{\sim}{r}}\times\hat{\underset{\sim}{x}})\left\{\frac{1}{(kr)}+\frac{i}{(kr)^2}\right\} . \tag{39}$$

The energy density in the dipole-field distribution can be calculated from (36) and (37) as

$$\Delta w_d = \frac{1}{2}\epsilon_0 \langle \underset{\sim}{E}_d^2 \rangle + \frac{1}{2}\mu_0 \langle \underset{\sim}{H}_d^2 \rangle$$

$$= \frac{1}{2}\mathrm{Re}\Big\langle \frac{\epsilon_0}{2}\left(\frac{1}{4\pi\epsilon_0}\right)^2 \sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k \int d^3k' \left(\frac{6\pi\epsilon_0 c^3\Gamma}{\omega^2}\right)\left(\frac{6\pi\epsilon_0 c^3\Gamma}{\omega'^2}\right)\left(\frac{\hbar\omega}{8\pi^3\epsilon_0}\right)^{1/2}\left(\frac{\hbar\omega'}{8\pi^3\epsilon_0}\right)^{1/2}$$

$$\times (\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{x}})(\hat{\underset{\sim}{\epsilon}}'\cdot\hat{\underset{\sim}{x}})|\underset{\sim}{G}|^2 \exp[-i(\omega-\omega')t + i\theta(\underset{\sim}{k},\sigma) - i\theta(\underset{\sim}{k}',\sigma')]\Big\rangle$$

$$+ \frac{1}{2}\mathrm{Re}\Big\langle \frac{\mu_0}{2}\left(\frac{c}{4\pi}\right)^2 \sum_{\sigma=1}^{2}\sum_{\sigma'=1}^{2}\int d^3k \int d^3k' \left(\frac{6\pi\epsilon_0 c^3\Gamma}{\omega^2}\right)\left(\frac{6\pi\epsilon_0 c^3\Gamma}{\omega'^2}\right)\left(\frac{\hbar\omega}{8\pi^3\epsilon_0}\right)^{1/2}\left(\frac{\hbar\omega'}{8\pi^3\epsilon_0}\right)^{1/2}$$

$$\times (\hat{\underset{\sim}{\epsilon}}\cdot\hat{\underset{\sim}{z}})(\hat{\underset{\sim}{\epsilon}}'\cdot\hat{\underset{\sim}{z}})|\underset{\sim}{F}|^2 \exp[-i(\omega-\omega')t + i\theta(\underset{\sim}{k},\sigma) - i\theta(\underset{\sim}{k}',\sigma')]\Big\rangle$$

$$= \frac{3\hbar c^3\Gamma^2}{16\pi^2}\int_0^{\omega_c}\frac{d\omega}{\omega}\left(|\underset{\sim}{F}|^2 + |\underset{\sim}{G}|^2\right) \quad . \qquad (40)$$

After appropriate averaging (including averaging over solid angle for a collection of randomly-oriented particle motions, and doubling to take into account the contributions of the two independent degrees of freedom in the model), the result reduces to a form which contains three terms (Puthoff (7)). The first is proportional to $1/r^2$, and constitutes the radiation field associated with the ZPF-driven dipole. As shown previously by Boyer (27), this radiation just replaces that being absorbed from the background, on a detailed-balance basis with regard to both frequency and angular distribution, and therefore does not result in an incremental change to the background. Of the two remaining (induction) field terms, a $1/r^4$ term predominates over a $1/r^6$ term at large distances, and is therefore the one of interest here. Designating this term by a prime, we have

$$\Delta w_d' = \frac{\hbar c \Gamma^2}{2\pi^2 r^4}\int_0^{\omega_c}\omega\, d\omega \quad , \qquad (41)$$

which leads to an overall spectral density

$$\Delta \rho_d'(\omega) = \frac{\hbar c \Gamma^2 \omega}{2\pi^2 r^4} \quad . \qquad (42)$$

Since according to (24) there is a relationship between Γ and the particle mass m for ZPF-driven fluctuation motion, the above can also be written

$$\Delta \rho_d'(\omega) = \frac{c^5 m^2 \omega}{2\hbar \omega_c^4 r^4} \quad . \tag{43}$$

ZPF-induced motion therefore leads to the generation of an electromagnetic field distribution in proximity to the mass that is proportional to frequency times mass squared, divided by r^4. According to (32), moreover, a field of just this form is required by the principle of equivalence. Eqns. (32) and (43) can therefore be equated to obtain the cutoff frequency (Shupe (28))

$$\omega_c = \sqrt{\frac{\pi c^5}{\hbar G}} \quad . \tag{44}$$

This result accords with a prediction by Sakharov (14), made on the basis of heuristic and dimensional arguments along general relativistic lines, that $\omega_c \sim \sqrt{c^5/\hbar G}$. In terms of the cutoff frequency ω_c, (44) can be inverted to yield the gravitational constant G in the form of a second Sakharov prediction, namely that the gravitational constant should be determined by an expression of the form $G \sim c^5/\hbar \int_0^{\omega_c} \omega\, d\omega$; specifically, we obtain

$$G = \frac{\pi c^5}{\hbar \omega_c^2} = \frac{\pi}{2} \frac{c^5}{\hbar \int_0^{\omega_c} \omega\, d\omega} \quad . \tag{45}$$

The main point, however, is that substitution of (44) into (29) yields Newton's Law with no adjustable parameters required,

$$\langle F \rangle = -\frac{G m^2}{R^2} \quad . \tag{46}$$

Thus, the gravitational interaction takes its place alongside the short-range van der Waals forces and the Casimir force as related phenomena which emerge from the underlying dynamics of the interaction of particles with the zero-point fluctuations of the vacuum electromagnetic field.

A major benefit of the approach developed herein is that it provides a basis for understanding various characteristics of the gravitational interaction hitherto unexplained. The relative weakness of the gravitational force under ordinary circumstances, for example, is due to the fact that the gravitational constant G, given by (45), reflects as the inverse square the high value of the ZPF cutoff frequency; the attractive nature of the force is simply a reflection of a property typical of van der Waals-type forces in general; the unipolar or single-valuedness of the "charge" (mass) can be traced to a positive-only *zitterbewegung* kinetic energy basis for the mass parameter; the fact that gravity cannot be shielded is a consequence of the fact that high-frequency quantum noise in general cannot be shielded, a factor which in other contexts sets a lower limit on the detectability of electromagnetic signals. In short, with a detailed theory in hand we are able to broaden our understanding of the gravitational interaction under various conditions, thereby enriching our knowledge of the mechanisms which underlie the gravitational force.

CONCLUSION

These studies of the ground state of hydrogen as a ZPF-determined state, and gravity as a ZPF force, indicate that the vacuum acts not as a passive background, but as an active dynamic plenum in determining the basic states of matter and their interaction, a concept that transcends the usual interpretation of the role and significance of the zero-point fluctuations of the background vacuum electromagnetic field.

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APPENDIX

The two-particle interaction potential based on the radiation-field van der Waals effect is given by (26), repeated here;

$$U = -\frac{\chi}{\mathcal{R}}\left(\frac{\sin\mathcal{R}}{\mathcal{R}}\right)^2 = -\frac{\chi}{2}\,\frac{1-\cos 2\mathcal{R}}{\mathcal{R}^3} \quad . \tag{A1}$$

As seen, this expression can be factored into two parts; one with a slow spatial variation, $1/\mathcal{R}$, and one with a rapid spatial variation (on the order of the Planck wavelength), $(\sin\mathcal{R}/\mathcal{R})^2$. Of interest in the gravitational interaction is not the rapidly-varying component, but rather an average value, averaged over a distance large compared to the Planck wavelength. With the potential given by (A1), the (normalized) force is given by

$$\mathcal{F} = -\frac{\partial U}{\partial \mathcal{R}} = \frac{\chi}{2}\frac{\partial}{\partial\mathcal{R}}\,\frac{1-\cos 2\mathcal{R}}{\mathcal{R}^3} \quad . \tag{A2}$$

As particle separation changes by an amount $\Delta\mathcal{R}$, the corresponding change in potential is given by

$$\Delta U = -\int_{\mathcal{R}_i}^{\mathcal{R}_i+\Delta\mathcal{R}} \mathcal{F}\, d\mathcal{R} = -\frac{\chi}{2}\int_{\mathcal{R}_i}^{\mathcal{R}_i+\Delta\mathcal{R}} \frac{\partial}{\partial\mathcal{R}}\,\frac{1-\cos 2\mathcal{R}}{\mathcal{R}^3}\, d\mathcal{R}$$

$$= -\frac{\chi}{2}\left[\frac{1-\cos 2(\mathcal{R}_i+\Delta\mathcal{R})}{(\mathcal{R}_i+\Delta\mathcal{R})^3} - \frac{1-\cos 2\mathcal{R}_i}{\mathcal{R}_i^3}\right] \quad . \tag{A3}$$

Assuming integration over a full cycle of the Planck variation so that $\cos 2(\mathcal{R}_i + \Delta\mathcal{R}) = \cos 2\mathcal{R}_i$, and recognizing that $\Delta\mathcal{R} \ll \mathcal{R}_i$ so that $(\mathcal{R}_i + \Delta\mathcal{R})^3 \approx \mathcal{R}_i^3 + 3\mathcal{R}_i^2\Delta\mathcal{R}$, we find that (A3) simplifies to

$$\Delta U = \frac{3\chi\Delta\mathcal{R}}{2}\,\frac{1-\cos 2\mathcal{R}_i}{\mathcal{R}_i^4} \quad . \tag{A4}$$

The change in potential, integrated over a cycle, is seen from (A4) to be sensitive to where in the cycle, $\theta_i = 2\mathcal{R}_i$, the integration was begun. The *average* change in potential is therefore determined by averaging over the range of possible initial starting points within the cycle, namely, $\pi n \le \mathcal{R}_i \le \pi(n+1)$, where n is an integer, $n \gg 1$.

By reference to standard Tables of Integrals (Dwight (29)) we find, using (A4),

$$\langle \Delta U \rangle = \frac{1}{\pi} \int_{\pi n}^{\pi(n+1)} \Delta U \, d\mathcal{R}_i$$

$$= -\frac{\chi \Delta \mathcal{R}}{2\pi} \left(\frac{1}{\mathcal{R}_i^3} \right)_{\pi n}^{\pi(n+1)} - \frac{2\chi \Delta \mathcal{R}}{\pi} \Bigg\{ -\frac{2\cos 2\mathcal{R}_i}{(2\mathcal{R}_i)^3} + \frac{\sin 2\mathcal{R}_i}{(2\mathcal{R}_i)^2} + \frac{\cos 2\mathcal{R}_i}{2\mathcal{R}_i}$$

$$+ \left[2\mathcal{R}_i - \frac{(2\mathcal{R}_i)^3}{3 \cdot 3!} + \frac{(2\mathcal{R}_i)^5}{5 \cdot 5!} - \cdots \right] \Bigg\}_{\pi n}^{\pi(n+1)} \quad . \qquad \text{(A5)}$$

Substitution of the limits of integration, with the recognition that $n \gg 1$ implies that $(n + 1)^p \approx n^p + pn^{p-1}$, then leads to

$$\langle \Delta U \rangle = \frac{\chi \Delta \mathcal{R}}{(\pi n)^2} - \frac{2\chi \Delta \mathcal{R}}{\pi n} \left[2\pi n - \frac{(2\pi n)^3}{3!} + \frac{(2\pi n)^5}{5!} - \cdots \right] . \qquad \text{(A6)}$$

But the term in brackets is recognized to be $\sin 2\pi n = 0$, so that $\langle \Delta U \rangle$ becomes

$$\langle \Delta U \rangle = \frac{\chi \Delta \mathcal{R}}{\mathcal{R}^2} \quad , \qquad \text{(A7)}$$

from which the average force can be calculated as

$$\langle \mathcal{F} \rangle = -\frac{\langle \Delta U \rangle}{\Delta \mathcal{R}} = -\frac{\chi}{\mathcal{R}^2} \quad . \qquad \text{(A8)}$$

The actual (unnormalized) force, $F = -\partial U/\partial R$, is recovered from the above with the aid of the definition for $\mathcal{R}$ following (26), yielding

$$\langle F \rangle = \frac{\omega_c}{c} \langle \mathcal{F} \rangle = -\frac{\hbar c \Gamma^2 \omega_c^2}{\pi R^2} \quad . \qquad \text{(A9)}$$

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