# Reply to "Comment on 'Gravity as a zero-point-fluctuation force' "

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Although mathematically self-consistent, Carlip's approach to the reanalysis of Sakharov gravity is flawed by the neglect of important physical constraints associated with the interaction, and leads to an incorrect $1/R^4$ spatial dependence for the force. When appropriate physical cutoffs are incorporated into the modeling, however, inverse-square-law Newtonian gravity emerges as originally derived.

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The author of the preceding [1] Comment (Carlip) has contributed a useful clarifying remark with regard to a mathematical averaging procedure used to evaluate a series in Ref. [2]. However, his sweeping conclusion (that it invalidates the quantum-fluctuation derivation of Newtonian gravity presented there) is *not* supported, and can be traced to his neglect of important physical factors that constrain the quantum-fluctuation interaction. When one follows through in detail the various facets of the issues raised by Carlip, one finds that the basic model and results of the original paper (Ref. [2]) still stand.

The core argument is as follows. In the quantum-fluctuation model for gravity presented in Ref. [2], the two-particle interaction potential, Eq. (56), derived from standard van der Waals theory, is of the form (neglecting a $\frac{9}{4}$ factor, removed later)

$$U=-\frac{\hbar c^2\Gamma^2}{\pi R^2}\int d\omega \sin\left[\frac{2R\omega}{c}\right], \tag{1}$$

where $\Gamma$ is a particle mass parameter, $R$ is the separation between the particles, and it is understood that the integral is over frequencies of relevance for the interaction. In [2] this expression was evaluated by use of an averaging procedure that involved integration over all frequencies, followed by extraction of leading terms in a series expansion as one often does for small-argument approximations. Taking into account the details of the development of the parameter $\Gamma$, shown to be proportional to the mass $m$, we found this to lead to the $U=-Gm^2/R$ form of Newtonian gravity.

The import of Carlip's criticism is that, from a strictly mathematical viewpoint, it is problematical to mix integration over all frequencies with the use of small-argument approximations that would in general be arguable only for very low frequencies (or small distances). There is some merit in this argument in that, in the absence of further consideration, higher-order terms in the series can cancel certain leading terms. Pursuing this approach he goes on, with consideration given only to mathematical concerns and without regard to physical constraints, to derive large-argument, asymptotic expressions which differ in the $R$ dependence from those in [2], yielding, for example, a $1/R^4$ dependence for force in the gravity model under consideration. This we show to be incorrect due to the neglect of important physics in the interaction under consideration.

The above apparently differing approaches and consequent results can be resolved in the following way. Key to the resolution is a relevant quote from a paper by Boyer [3] in which he discusses the pertinent physics and mathematics involved in the van der Waals–type interaction potential shown above: "However, when the polarizable particles are very far apart, it is only the very low frequencies of oscillation which we expect to play a significant role. For high frequencies (small wavelengths), there will be cancelling effects between adjacent frequencies if the particles are very far apart, since the slight phase shifts will build up over the large distances. Thus in the asymptotic region $R\rightarrow\infty$, we expect only the frequencies near $\omega\approx 0$ to affect the interaction of polarizable particles. *Although the expressions we employ are conveniently expressed in terms of integrals over all frequencies, only the low-frequency parts actually contribute in the asymptotic region of large separation*" (italics mine). Thus, although characterized by Carlip as "*ad hoc*," the restriction to low-frequency contributions in the interaction potential (typically handled by the use of cutoffs, damping factors, or resonance denominators) is a key feature throughout the literature in the treatment of van der Waals–type interactions.

Therefore, returning to (1) above, and being very explicit with regard to low-frequency behavior, we rewrite (1) in its original detailed form referenced in [2] as Ref. [27], and given here as [4],

$$U=-\frac{\hbar c^2\Gamma^2}{\pi R^2}\int_0^{\omega_i}\frac{d\omega\,\omega^4\sin\left[\frac{2R\omega}{c}\right]}{(\omega_0^2-\omega^2)^2}, \tag{2}$$

where $\omega_0\rightarrow 0$ in the resonance denominator corresponds to the (vanishing) binding energy for an asymptotically free parton (quark), and $\omega_i\rightarrow 0$ corresponds to the low-frequency cutoff of meaningful, non-phase-cancelling interactions at large separations as outlined above by Boyer (not to be confused with the high-frequency cutoff of the background fluctuations themselves).

With $\omega_i$ taken as the interaction cutoff limit at large separation, it can be assumed that as $\omega$ approaches $\omega_0$ from the low-frequency side the increased resonance response specifically determines (constrains) $\omega_i$ to $\omega_i \approx \omega_0 - \to 0$; that is, $\omega_i$ is constrained to lie slightly below $\omega_0$ as both reduce to zero. As a result, between the low-frequency resonance and the explicit, corollary low-frequency interaction cutoff, a small-argument approximation $\sin(2R\omega/c) \approx (2R\omega/c)$ is certainly justified, in which case (2) is reduces to

$$U = -\frac{2\hbar c \Gamma^2}{\pi R} \int_0^{\omega_i} \frac{d\omega\, \omega^5}{(\omega_0^2 - \omega^2)^2}$$
$$= -\frac{2\hbar c \Gamma^2}{\pi R} \frac{\omega_0^4}{2(\omega_0^2 - \omega_i^2)}, \quad (3)$$

where in the integration we have kept only the dominant (resonance) term. This expression further reduces to

$$U = -\frac{Gm^2}{R} \left[ \frac{(\omega_0/\omega_c)^2}{1 - \frac{(\omega_i/\omega_c)^2}{(\omega_0/\omega_c)^2}} \right] \to -\frac{Gm^2}{R}, \quad (4)$$

where we have substituted $\Gamma = Gm/c^3$ [Eq. (23), Ref. [2]]; we have normalized frequencies to $\omega_c = (\pi c^5/\hbar G)^{1/2}$, the background zero-point-energy Planck cutoff proposed by Sakharov and derived from first principles in Ref. [2]; and, in the low-frequency limit where $\omega_i \to \omega_0 -$ and $\omega_0 \to 0$, we let $\omega_0 \to 0$ as $(\omega_0/\omega_c)^2 = \delta \to 0$, and $\omega_i \to \omega_0 -$ as $(\omega_i/\omega_c)^2 = (\omega_0/\omega_c)^2(1-\delta)$, to derive the Newtonian form.

This brief exposition is admittedly heuristic, depending as it does on a mix of physical argument and mathematical approximation, and clearly deserves more explication than can be given in a brief note. Nonetheless, from such studies it appears inescapable that in the low-frequency interaction regime, which surely obtains for widely separated particles, there is a weak, residual van der Waals–like inverse-square-law attractive force whose dependence on mass and the gravitational constant, derived by independent means (Ref. [2]), recommends it as a viable candidate for a quantum-fluctuation model of the gravitational interaction.

Now with regard to the physical processes that underpin the quantum-fluctuation model, Carlip makes a number of statements that are simply incorrect. For example, in referring to the interaction of charged pointlike particles (partons) with the background stochastic electromagnetic zero-point-fluctuation fields, Carlip avers that "it is easy to see that these cannot simply be the ordinary constituent quarks—mass ratios come out wrong... ." To the contrary, as detailed in Ref. [2], the partons *are indeed* the constituent quarks: their respective masses $m$ are expressed in terms of the parameter $\Gamma$, wherein the definition $\Gamma = Gm/c^3 = q^2/6\pi\epsilon_0 m_0 c^3$ depends explicitly, for each particle, on an adjustable bare mass parameter $m_0$, thus assuring that correct mass ratios can obtain.

In yet further discussion of the parton concept, Carlip expresses the opinion that since "quarks and leptons are observed to be pointlike to scales of $10^{-17}$ cm ... only wavelengths smaller than this should contribute to Puthoff's gravitational interaction." Again, to the contrary, as with all van der Waals–type interactions, whether at the molecular, atomic, or quark and lepton scale, the wavelengths involved in interparticle interaction are *greater*, not smaller, than the particles involved, as is abundantly clear in the Boyer excerpt quoted above. (Such is true, by the way, for most other particle-field interactions as well, e.g., at radio frequencies, light, etc.) Thus, although individual particles can respond to high-frequency fluctuations (indeed precisely because of their pointlike nature), and this fluctuation energy does contribute to their individual masses (as per standard QED calculations), the *interaction* potential of two or more particles depends only on their *correlated* motions, regarding which only the low-frequency, long-wavelength components contribute in the asymptotic region of large separation, again as detailed by Boyer.

In short, the interaction of point-like particles with the background zero-point-fluctuation fields, and the particle-particle interaction potentials resulting therefrom, are precisely the reverse of Carlip's suppositions. As a result, conclusions drawn by Carlip, such as, e.g., that long-wavelength fluctuations would "see neutrons as neutral, and thus massless, particles," are invalid. Long-wavelength electromagnetic fluctuations would interact with the charged pointlike constituents of neutrons (quarks) just as long-wavelength radio waves interact with charged pointlike electrons. Finally, Carlip suggests that binding energy contributions to mass would appear to be neglected in vacuum-fluctuation models of mass generation. In drawing this inference, however, he overlooks the fact that such binding energies act as constraints on particle motion response to the fluctuation field (as is evident in standard treatments of, say, harmonic oscillator response to such fields) and would thereby alter the mass-generating function accordingly.

In summary, we find that detailed consideration of the quantum-fluctuation model for inverse-square-law Newtonian gravity presented in Ref. [2] with the interactions treated mathematically in terms of the explicit display of appropriate low-frequency behavior and cutoffs, leads to the originally derived result without the apparent ambiguities noted by Carlip. Thus, contrary to Carlip's assertion, the basic model and results of the original paper (Ref. [2]), which detail a vacuum-fluctuation underpinning for gravitational phenomena in accordance with a model originally proposed by Sakharov, still stand.

[1] S. Carlip, preceding Comment, Phys. Rev. A **47**, 3452 (1992).
[2] H. E. Puthoff, Phys. Rev. A **39**, 2333 (1989).
[3] T. H. Boyer, Phys. Rev. A **5**, 1799 (1972).
[4] T. H. Boyer, Phys. Rev. A **7**, 1832 (1973), Eq. (92).