Engineering the Zero-Point Field and Polarizable Vacuum for Interstellar Flight

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A theme that has come to the fore in advanced planning for long-range space exploration is the concept of "propellantless propulsion" or "field propulsion". One version of this concept involves the projected possibility that empty space itself (the quantum vacuum, or space-time metric) might be manipulated so as to provide energy/thrust for future space vehicles [1]. Although far reaching, such a proposal is solidly grounded in modern theory that describes the vacuum as a polarizable medium that sustains energetic quantum fluctuations. Thus the possibility that matter/vacuum interactions might be engineered for space-flight applications is not a priori ruled out, although certain constraints need to be acknowledged. The structure and implications of such a far-reaching hypothesis are considered herein.

Keywords: Zero-point energy, warp drive, propellantless propulsion, metric engineering, interstellar flight

1. Introduction

The concept of "engineering the vacuum" found its first expression in the mainstream physics literature when it was introduced by T. D. Lee in his textbook Particle Physics and Introduction to Field Theory [2]. There he stated: "The experimental method to alter the properties of the vacuum may be called vacuum engineering... If indeed we are able to alter the vacuum, then we may encounter some new phenomena, totally unexpected." This legitimization of the vacuum engineering concept was based on the recognition that the vacuum is characterized by parameters and structure that leave no doubt that it constitutes an energetic medium in its own right. Foremost among these are its properties that (1) within the context of quantum theory the vacuum is the seat of energetic particle and field fluctuations, and (2) within the context of general relativity the vacuum is the seat of a space-time structure (metric) that encodes the distribution of matter and energy. Indeed, on the flyleaf of a book of essays by Einstein and others on the properties of the vacuum we find the statement "The vacuum is fast emerging as the central structure of modern physics" [3].

Given the known characteristics of the vacuum, one might reasonably inquire as to why it is not immediately obvious how to catalyze robust interactions of the type sought for space-flight applications. To begin, in the case of quantum fluctuations there are uncertainties that remain to be clarified regarding global thermodynamic and energy constraints. Furthermore, the energetic components of potential utility involve very small-wavelength, high-frequency fields and thus resist facile engineering solutions. With regard to perturbation of the space-time metric, the required energy densities exceed by many orders of magnitude values achievable with existing engineering techniques. Nonetheless, we can examine the constraints, possibilities and implications under the expectation that as technology matures, felicitous means may be found that permit the exploitation of the enormous, as-yet-untapped potential of so-called "empty space".

2. Propellantless Propulsion

2.1 Global Constraint

Regardless of the mechanisms that might be entertained with regard to "propellantless" or "field" propulsion of a spaceship, there exist certain constraints that can be easily overlooked but must be taken into consideration. A central one is that, because of the law of conservation of momentum, the center of mass-energy (CM) of an initially stationary isolated system cannot change its position if not acted upon by outside forces. This means that propellantless or field propulsion, whatever form it takes, is constrained to involve coupling to the external universe in such a way that the displacement...
of the CM of the spaceship is matched by a counte-
ring effect in the universe to which it is cou-
pled, so as not to violate the global CM constraint. 
Therefore, before one launches into a detailed in-
vestigation of a proposed propulsion mechanism it
is instructive to apply this principle as an overall
constraint to determine whether the principle is vio-
lated. Surprising subtleties may be involved in such
an assessment, as illustrated in the following exam-
ple.

2.2 An Example: “ExH” 
Electromagnetic Field Propulsion

A recurring theme in electromagnetic propulsion
considerations is that one might employ crossed
electric and magnetic fields to generate propulsive
force, what we might call ExH propulsion. The idea
is based on the fact that propagating electromagnetic fields (photons) possess momentum carried
by the crossed (orthogonal) E and H fields (Poynting
vector). This raises the issue as to whether static
(i.e., non-propagating) ExH fields also constitute
momentum (as the mathematics would imply), and in
particular whether changes in static fields could
result in the transfer of momentum to an attached
structure. As it turns out, the answer can be yes, as
illustrated in the example of the Feynman disk para-
dox [4]. Electric charge distributed around the rim
of a non-rotating disk generates a static electric
field that extends outward from the rim, and a cur-
rent-carrying coil of wire mounted perpendicular to
the plane of the disk generates a static dipole mag-
netic field. The two fields result in a static ExH
distribution that encircles the disk. Even though noth-
ing is apparently in motion, if we take the ExH mo-
mentum concept seriously it would appear that there
is angular momentum “circulating” about the disk in
the static fields. That this is in fact the case is dem-
onstrated by the fact that when the current in the
coil is interrupted, thereby extinguishing the mag-
netic field component of the ExH distribution, the
disk begins to rotate. This behaviour remains even at a
temperature of absolute zero. Such a concept is almost certain to have pro-
found implications for future space travel, as we
will now discuss.

Pursuit of the linear thrust possibility, however,
leads one to a rich literature concerning so-called
“hidden momentum” that, perhaps surprisingly, de-
nies this possibility [6]. The “hidden momentum”
phrase refers to the fact that although the linear
ExH fields do carry momentum as in the angular
case, the symmetry conditions for the linear case
are such that there exists a cancelling mechanical
momentum contained in the structures even though
a structure’s CM itself is stationary (see Appendix A).
Specifically, it can be shown on very general grounds
that, contrary to the case for angular momentum
(e.g., the Feynman disk), the total linear momentum
of any stationary distribution of matter, charge and
their currents, and their associated fields, must van-
ish. In other words, barring a new discovery that
modifies the present laws of physics, any such dis-
tribution cannot generate a propulsive force with-
out emitting some form of reaction mass or energy,
or otherwise imparting momentum to another sys-
tem [7].

3. The Quantum Vacuum

3.1 Zero-Point Energy (ZPE) Background

Quantum theory tells us that so-called “empty space”
is not truly empty, but is the seat of myriad ener-
getic quantum processes. Specifically, quantum field
theory tells us that, even in empty space, fields
e.g., the electromagnetic field) continuously fluctu-
ate about their zero baseline values. The energy
associated with these fluctuations is called zero-
point energy (ZPE), reflecting the fact that such
activity remains even at a temperature of absolute zero. Such a concept is almost certain to have pro-
found implications for future space travel, as we
would now discuss.

When a hypothetical ZPE-powered spaceship
strains against gravity and inertia, there are three
elements of the equation that the ZPE technology
could in principle address: (1) a decoupling from
gravity, (2) a reduction of inertia, or (3) the genera-
tion of energy to overcome both.

3.2 Gravity

With regard to a ZPE basis for gravity, the Russian
physicist Andrei Sakharov was the first to propose
that in a certain sense gravitation is not a funda-
mental interaction at all, but rather an induced ef-
fect brought about by changes in the quantum-fluc-
tuation energy of the vacuum when matter is present
[8]. In this view, the attractive gravitational force is
more akin to the induced van der Waals and Casimir
forces, than to the fundamental Coulomb force. Al-
though quite speculative when first introduced by
Sakharov in 1967, this hypothesis has led to a rich
literature on quantum-fluctuation-induced gravity.
(The latter includes an attempt by one of the au-
thors to flesh out the details of the Sakharov proposal [9], though difficulties remain [10]. Given the possibility of a deep connection between gravity and the zero-point fluctuations of the vacuum, it would therefore appear that a potential route to gravity decoupling would be via control of vacuum fluctuations.

### 3.3 Inertia

Closely related to the ZPE basis for gravity is the possibility of a ZPE basis for inertia. This is not surprising, given the empirical fact that gravitational and inertial masses have the same value, even though the underlying phenomena are quite disparate; one is associated with the gravitational attraction between bodies, while the other is a measure of resistance to acceleration, even far from a gravitational field. Addressing this issue, the author and his colleagues evolved a ZPE model for inertia which developed the concept that although a uniformly moving body does not experience a drag force from the (Lorentz-invariant) vacuum fluctuations, an accelerated body meets a resistive force proportional to the acceleration [11], an approach that has had a favourable reception in the scientific community [12]. Again, as in the gravity case, it would therefore appear that a potential route to the reduction of inertial mass would be via control of vacuum fluctuations.

Investigation into this possibility by the U.S. Air Force’s Advanced Concepts Office at Edwards Air Force Base resulted in the generation of a report entitled Mass Modification Experiment Definition Study that addressed just this issue [13]. Included in its recommendations was a call for precision measurement of what is called the Casimir force. The Casimir force is an attractive quantum force between closely spaced metal or dielectric plates (or other structures) that derives from partial shielding of the interior region from the background zero-point fluctuations of the vacuum electromagnetic field, which results in unbalanced ZPE radiation pressures [14]. Since issuance of the report, such precision measurements have been made which confirm the Casimir effect to high accuracy [15], measurements which even attracted high-profile attention in the media [16]. The relevance of the Casimir effect to our considerations is that it constitutes experimental evidence that vacuum fluctuations can be altered by technological means. This suggests the possibility that, given the models discussed, gravitational and inertial masses might also be amenable to modification. The control of vacuum fluctuations by the use of cavity structures has already found practical application in the field of cavity quantum electrodynamics, where the spontaneous emission rates of atoms are subject to manipulation [17]. Therefore, it is not unreasonable to contemplate the possibility of such control in the field of space propulsion.

### 3.4 Energy Extraction

With regard to the extraction of energy from the vacuum fluctuation energy reservoir, there are no energetic or thermodynamic constraints preventing such release under certain conditions [18]. And, in fact, there are analyses in the literature that suggest that such mechanisms are already operative in nature in the “powering up” of cosmic rays [19], or as the source of energy release from supernovas [20] and gamma-ray bursts [21].

For our purposes, the question is whether the ZPE can be “mined” at a level practical for use in space propulsion. Given that the ZPE energy density is conservatively estimated to be on the order of nuclear energy densities or greater [22], it would constitute a seemingly ubiquitous energy supply, a veritable “Holy Grail” energy source.

One of the first researchers to call attention to the principle of the use of the Casimir effect as a potential energy source was Robert Forward at Hughes Research Laboratories in Malibu, CA [23]. Though providing “proof-of-principle,” unlike the astrophysical implications cited above the amount of energy release for mechanical structures under laboratory conditions is minuscule. (The collapse of a pair of one-centimeter-square Casimir plates from, say, 2 microns to 1 micron in 1 microsecond, generates around 1/10 microwatt.) In addition, the conservative nature of the Casimir effect would appear to prevent recycling, though there have been some suggestions for getting around this barrier [24]. Alternatives involving non-recycling behaviour, such as plasma pinches [25] or bubble collapse in sonoluminescence [26], have been investigated in our laboratory and elsewhere, but as yet without real promise for energy applications.

Vacuum energy extraction approaches by other than the Casimir effect are also being considered. One approach that emerged from the Air Force’s Mass Modification... study [13] was the suggestion that the ZPE-driven cosmic ray model be explored under laboratory conditions to determine whether protons could be accelerated by the proposed cosmic ray mechanism in a cryogenically-cooled, collision-free vacuum trap. Yet another proposal (for
which a patent has been issued) is based on the concept of beat-frequency downshifting of the more energetic high-frequency components of the ZPE, by use of slightly detuned dielectric-sphere antennas [27].

In our own laboratory we have considered an approach based on perturbation of atomic or molecular ground states, hypothesized to be equilibrium states involving dynamic radiation/absorption exchange with the vacuum fluctuations [28]. In this model atoms or molecules in a ZPE-limiting Casimir cavity are expected to undergo energy shifts that would alter the spectroscopic signatures of excitations involving the ground state. We have initiated experiments at a synchrotron facility to explore this ZPE/ground-state relationship, though so far without success. In addition to carrying out experiments based on our own ideas, our laboratory also acts as a clearing-house to evaluate the experimental concepts and devices of others who are working along similar lines. Details can be found on our website, www.earthtech.org.

Whether tapping the ZPE as an energy source or manipulating the ZPE for gravity/inertia control are but gleams in a spaceship designer’s eye, or a Royal Road to practical space propulsion, is yet to be determined. Only by explorations of the type described here will the answer emerge. In the interim a quote by the Russian science historian Roman Podolny would seem to apply: “It would be just as presumptuous to deny the feasibility of useful application as it would be irresponsible to guarantee such application” [29].

4. The Space-Time Metric (“Metric Engineering” Approach)

Despite the apparently daunting energy requirements to perturb the space-time metric to a significant degree, we examine the structure that such perturbations would take under conditions useful for space-flight application, a “Blue Sky” approach, as it were.

Although topics in general relativity are routinely treated in terms of tensor formulations in curved space-time, we shall find it convenient for our purposes to utilize one of the alternative methodologies for treating metric changes that has emerged over the years in studies of gravitational theories. The approach, known as the polarizable vacuum (PV) representation of general relativity (GR), treats the vacuum as a polarizable medium [30]. The PV approach treats metric changes in terms of the permittivity and permeability constants of the vacuum, $\varepsilon_0$ and $\mu_0$, essentially along the lines of the “THFL” methodology used in comparative studies of gravitational theories [31]. Such an approach, relying as it does on parameters familiar to engineers, can be considered a “metric engineering” approach.

In brief, Maxwell’s equations in curved space are treated in the isomorphism of a polarizable medium of variable refractive index in flat space [32]; the bending of a light ray near a massive body is modelled as due to an induced spatial variation in the refractive index of the vacuum near the body; the reduction in the velocity of light in a gravitational potential is represented by an effective increase in the refractive index of the vacuum, and so forth. As elaborated in Ref. 30 and the references therein, though differing in some aspects from GR, PV modelling can be carried out for cases of interest in a self-consistent way so as to reproduce to appropriate order both the equations of GR, and the match to the classical experimental tests of those equations.

Specifically, the PV approach treats such measures as the velocity of light, the length of rulers (atomic bond lengths), the frequency of clocks, particle masses, and so forth, in terms of a variable vacuum dielectric constant $K$ in which vacuum permittivity $\varepsilon_0$ transforms to $\varepsilon_0 \rightarrow KE_0$, vacuum permeability to $\mu_0 \rightarrow K\mu_0$. In a planar or solar gravitational potential $K = 1 + 2GM/rc^2 > 1$, and the results are as shown in Table 1. Thus, the velocity of light is reduced, light emitted from an atom is redshifted as compared with an atom at infinity ($K = 1$), rulers shrink, etc.

As one example of the significance of the tabulated values, the dependence of fundamental length measures (ruler shrinkage) on the variable $K$ indicates that the dimensions of material objects adjust in accordance with local changes in vacuum polarizability - thus there is no such thing as a perfectly rigid rod. From the standpoint of the PV approach this is the genesis of the variable metric that is of such significance in GR studies. It also permits us to define, from the viewpoint of the PV approach, just what precisely is meant by the label “curved space.” In the vicinity of, say, a planet or star, where $K > 1$, if one were to take a ruler and measure along a radius vector $R$ to some circular orbit, and then measure the circumference $C$ of that orbit, one would obtain $C < 2\pi R$ (as for a concave curved surface). This is a consequence of the ruler being relatively shorter during the radial measuring proc-
ess when closer to the body where $K$ is relatively greater, as compared
to its length during the circumfer-
tential measuring process when fur-
ther from the body. Such an in-
fluence on the measuring process due
to induced polarizability changes in
the vacuum near the body leads to
the GR concept that the presence of the body “influences the metric,”
and correctly so.

We are now in a position to con-
sider application of this “metric en-
ingineering” formalism to the type of
questions relevant to space propul-
sion. As we show in Appendix B,
under certain conditions the metric
can in principle be modified to re-
duce the value of the vacuum di-
electric constant $K$ to below unity.
Returning to Table 1, we see that a
$K < 1$ solution permits the addition
of another column for which the des-
criptors are reversed, as shown in Table 2.

Under such conditions of extreme space-time
perturbation, the local velocity of light (as seen from a
reference frame at infinity) is increased, mass de-
creases, energy bond strengths increase, etc., features
presumably attractive for interstellar travel.

As an example, one specific approach that has
generated considerable commentary in the technical
literature is the so-called Alcubierre Warp Drive,
named after its creator, general relativity theorist
Miguel Alcubierre [33, 34]. Alcubierre showed that
by distorting the local space-time metric in the re-

<table>
<thead>
<tr>
<th>Variable</th>
<th>Determining Equation</th>
<th>$\lambda &gt; 1$ (typical mass distribution, $M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity of light $v_L(K)$</td>
<td>$v_L = c/K$</td>
<td>velocity of light $&lt; c$</td>
</tr>
<tr>
<td>mass $m(K)$</td>
<td>$m = m_0 K^{1/2}$</td>
<td>effective mass increases</td>
</tr>
<tr>
<td>frequency $\omega(K)$</td>
<td>$\omega = \omega_0 / \sqrt{K}$</td>
<td>redshift toward lower frequencies</td>
</tr>
<tr>
<td>time interval $\Delta t(K)$</td>
<td>$\Delta t = \Delta t_0 / \sqrt{K}$</td>
<td>clocks run slower</td>
</tr>
<tr>
<td>energy $E(K)$</td>
<td>$E = E_0 / \sqrt{K}$</td>
<td>lower energy states</td>
</tr>
<tr>
<td>length dim. $L(K)$</td>
<td>$L = L_0 / \sqrt{K}$</td>
<td>objects shrink</td>
</tr>
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<table>
<thead>
<tr>
<th>Variable</th>
<th>Determining Equation</th>
<th>$\lambda &lt; 1$ (engineered metric)</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity of light $v_L(K)$</td>
<td>$v_L = c/K$</td>
<td>velocity of light $&gt; c$</td>
</tr>
<tr>
<td>mass $m(K)$</td>
<td>$m = m_0 K^{1/2}$</td>
<td>effective mass decreases</td>
</tr>
<tr>
<td>frequency $\omega(K)$</td>
<td>$\omega = \omega_0 / \sqrt{K}$</td>
<td>blueshift toward higher frequencies</td>
</tr>
<tr>
<td>time interval $\Delta t(K)$</td>
<td>$\Delta t = \Delta t_0 / \sqrt{K}$</td>
<td>clocks run faster</td>
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<td>$E = E_0 / \sqrt{K}$</td>
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</tr>
<tr>
<td>length dim. $L(K)$</td>
<td>$L = L_0 / \sqrt{K}$</td>
<td>objects expand</td>
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</tbody>
</table>

When it comes to engineering the Alcubierre
solution, however, seemingly insurmountable
obstacles emerge. For a 100 m warp bubble the
bubble wall thickness approaches a Planck length
($\sim 10^{-35}$ m) and the (negative) energy required
is roughly 10 orders of magnitude greater than the
total mass of the universe! [35] Further theoretical
effort has resulted in a reduction of the energy
requirement to somewhat below a solar mass, an
impressive advance but still quite impractical [36].
Analysis of related alternatives such as the Krasnikov
Tube [37] and traversable wormholes have fared no
better [38]. Thus, if success is to be achieved, it
must rest on some as yet unforeseen breakthrough
about which we can only speculate, such as a tech-
ology to cohere otherwise random vacuum fluct-
uation energy.

Clearly then, calculations for the proposed
geometries are by no means directly applicable to
the design of a space propulsion drive. However,
these sample calculations indicate the direction of
potentially useful trends derivable on the basis of
the application of GR principles as embodied in a
metric engineering approach, with the results con-
strained only by what is achievable practically in an
engineering sense. The latter is, however, a daunting
constraint.

5. Conclusions

In this paper we have touched briefly on innovative
forms of space propulsion, especially those that
might exploit properties of the quantum vacuum or
the space-time metric in a fundamental way. At this
point in the development of such nascent concepts
it is premature to even guess at an optimum strategy, let alone attempt to forge a critical path; in fact, it remains to be determined whether such exploitation is even feasible. Nonetheless, only by inquiring into such concepts in a rigorous way can we hope to arrive at a proper assessment of the possibilities and thereby determine the best course of action to pursue in our steps first to explore our solar system environment, and then one day to reach the stars.

Appendix A - Hidden Momentum

Consider a stationary current loop which consists of an incompressible fluid of positive charge density $\rho$ circulating at velocity $v$ clockwise around a loop of non-conductive piping of cross sectional area $a$. The loop is immersed in a constant uniform electric field $E$.

The magnetic field created by the current loop combines with the electric field to produce an electromagnetic field momentum given by

$$p_{EM} = \frac{1}{c^2} \int E \times H \, dV$$  \hspace{1cm} (A1)

However, in steady state situations, this is equal to (Ref. 6)

$$p_{EM} = \frac{1}{c^2} \int J \phi \, dV$$  \hspace{1cm} (A2)

With reference to the above figure, the only non-zero component of momentum surviving this integration is directed horizontally across the page. Using the expression Eq. (A2), this computes to

$$p_{EM} = \frac{Iw}{c^2} \left( \psi_{\text{top}} - \psi_{\text{bottom}} \right) = \frac{IEhw}{c^2}$$  \hspace{1cm} (A3)

where the sense is from left to right. From this it is concluded that there is a steady net linear momentum stored in the electromagnetic fields. We will now show there is another momentum, equal and opposite to this electromagnetic field momentum.

Since the current flowing in the loop is given by $I = \rho av$, the velocity of the fluid is everywhere $v = I/\rho a$. Meanwhile, the external electric field $E$ creates a pressure difference between the bottom and the top of the fluid given by $P = \rho Eh$. Moving to the left, therefore, is a net energy flux $S$ (energy per unit area per unit time) given by

$$S = P_v = (\rho Eh) \times (I/\rho a) = \frac{IEh}{a}$$  \hspace{1cm} (A4)

But since energy has mass, Eq. (A4) may be converted to an expression for momentum. This is mostly accomplished by writing the Einstein relation $E = mc^2$ in flux density form as $S = gc^2$, where $g$ is the momentum per unit volume. It now follows that, due to the different pressures at the top and bottom of the loop, there must be a net overall momentum directed to the left given by

$$p_{\text{mech}} = gaw = \frac{Saw}{c^2} = \frac{IEhw}{c^2}$$  \hspace{1cm} (A5)

where the subscript ‘mech’ draws attention to the apparently entirely mechanical origin of this momentum.

Eqs. (A5) and (A3) demonstrate that the electromagnetic momentum is balanced by an equal and opposite mechanical momentum. Because of its rather obscure nature, this momentum has been referred to in the literature as “hidden momentum”. This is a particular example of the general result that a net static linear field momentum will always be balanced by an equal and opposite hidden mechanical momentum. In practical terms, this means that the creation of linear field momentum cannot give rise to motion because the field momentum is automatically neutralized by a mechanical momentum hidden within the structure, so that the whole system remains stationary. This inability to utilize linear field momentum for propulsion is guaranteed by the law of momentum conservation.

Appendix B - Metric Engineering Solutions

In the polarizable vacuum (PV) approach the equa-
tion that plays the role of the Einstein equation (curvature driven by the mass-energy stress tensor) for a single massive particle at the origin is (Ref. 30)

\[ \nabla^2 \sqrt{K} - \frac{1}{(c/ K)^2} \frac{\partial^2 \sqrt{K}}{\partial t^2} = \]

\[ - \sqrt{K} \left[ \frac{\hbar w^2}{2\sqrt{K}} \right] \delta^3 \left( \frac{\lambda}{K} \right) + \frac{1}{2} \left( \frac{b^2}{K} + K \frac{\partial}{\partial K} \right) \]

\[ - \frac{\lambda}{K^2} \left[ (\nabla K)^2 + \frac{1}{(c/ K)^2} \left( \frac{\partial K}{\partial t} \right)^2 \right] \]

(B1)

where

\[ w = v (c/ K) \]

In this PV formulation of GR, changes in the vacuum dielectric constant \( \kappa \) are driven by mass density (first term), EM energy density (second term), and the vacuum polarization energy density itself (third term). (The constant \( \lambda = e^4 / 32\pi G \), where \( G \) is the gravitational constant.)

In space surrounding an uncharged spherical mass distribution (e.g., a planet) the static solution (\( \partial K / \partial t = 0 \)) to the above is found by solving

\[ \frac{d^2 \sqrt{K}}{dr^2} + 2 \frac{d \sqrt{K}}{r \, dr} = \frac{1}{\sqrt{K}} \left( \frac{d \sqrt{K}}{dr} \right)^2 \]

(B2)

The solution that satisfies the Newtonian limit is given by

\[ K = \left( \frac{\sqrt{K}}{\kappa} \right) = e^{2GM/r c^2} = 1 + 2 \left( \frac{GM}{rc^2} \right) + ... \] (B3)

which can be shown to reproduce to appropriate order the standard GR Schwarzschild metric properties as they apply to the weak-field conditions prevailing in the solar system.

For the case of a mass \( M \) with charge \( Q \), the electric field appropriate to a charged mass imbedded in a variable-dielectric-constant medium is given by

\[ \oint d\mathbf{a} = K E (4\pi r^2) = Q \] (B4)

which leads to (for spherical symmetry, with \( b^2 = Q^2 G^4 \pi K c^4 \))

\[ \frac{d^2 \sqrt{K}}{dr^2} + 2 \frac{d \sqrt{K}}{r \, dr} = \frac{1}{\sqrt{K}} \left( \frac{d \sqrt{K}}{dr} \right)^2 - \frac{b^2}{r^4} \] (B5)

which should be compared with Eq. (B2). The solution here as a function of charge (represented by \( b \)) and mass (represented by \( a = GM/c^2 \)) is given by

\[ \sqrt{K} = \cosh \left( \frac{\sqrt{a^2 - b^2}}{r} \right) + \frac{a}{\sqrt{a^2 - b^2}} \sinh \left( \frac{\sqrt{a^2 - b^2}}{r} \right) \]

for

\[ a^2 > b^2 \] (B6)

For the weak-field case above reproduces the familiar Reissner-Nordström metric [39]. For \( b^2 > a^2 \), however, the hyperbolic solutions turn trigonometric, and \( K \) can take on values \( K < 1 \).

References

7. Proposals to push directly against the space-time metric or quantum vacuum, i.e., use the rest of the Universe as a springboard by means presently unknown, fall into the latter category.
10. H.E. Puthoff, “Reply to Comment on “Gravity as a zero-

(Received 2 August 2001)