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An Assessment of Faster-Than-Light Spacetimes: Make or Break Issues

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Implementation of faster-than-light (FTL) interstellar travel via traversable wormholes, warp drives, or other spacetime modification schemes generally requires the engineering of spacetime into very specialized local geometries. The analysis of these via Einstein's General Theory of Relativity plus the resultant equations of state demonstrates that such geometries require the use of "exotic" matter. It has been claimed that since such matter violates the energy conditions FTL spacetimes are not plausible. However, it has been shown that this is a spurious issue. The identification, magnitude, and production of exotic matter is seen to be a key technical challenge, however. FTL spacetimes also possess features that challenge the notions of causality, and there are alleged constraints placed upon them by quantum effects. These issues are reviewed and summarized, and an assessment on the present state of their resolution is provided.

Nomenclature

AU	=	astronomical unit (mean Earth-Sun distance)
B_c	=	critical quantum electrodynamic (vacuum breakdown) magnetic field intensity
$b(r)$	=	wormhole shape function
c	=	speed of light
χ_e	=	Euler Number
D	=	dimension of spacetime
d	=	cavity wall or plate separation distance
d_f	=	number of degrees of freedom per spatial point
$d\Omega$	=	solid angle element
ds^2	=	invariant distance function (or metric) in spacetime
δ	=	phase of squeezing
Δ	=	warp bubble wall thickness
E_c	=	critical quantum electrodynamic (vacuum breakdown) electric field intensity
E_{warp}	=	energy required to build a warp bubble
E_{wh}	=	energy required to build a traversable wormhole
e	=	electron charge
$f(r_{sh})$	=	warp bubble shape function
G	=	universal gravitation constant
g	=	topological genus
$g_{\oplus}$	=	acceleration of gravity at the Earth's surface
g_{metric}	=	matrix determinant of spacetime metric
$\hbar$	=	Planck's reduced constant
j	=	energy eigenvalue of zero-point fluctuation field mode
K^n	=	n-dimensional Klein bottle
L	=	length
λ	=	wavelength of zero-point fluctuation field mode
M	=	mass of astronomical body
$M_{\oplus}$	=	mass of Earth
M_J	=	mass of Jupiter

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M_{ship} = mass of spaceship
M_{warp} = equivalent mass required to build a warp bubble
M_{wh} = equivalent mass required to build a traversable wormhole
m_e = electron mass
ω = angular frequency
p_i = pressures in matter or in a quantum field
p_r = radial pressure
φ = spherical azimuth angle
$\phi(r)$ = wormhole redshift function
R = radius of warp bubble
$R_\oplus$ = radius of Earth
R_{ship} = size of spaceship
$\Re^n$ = n-dimensional real line
r = radial distance
r_S = Schwarzschild radius of astronomical body
r_{sh} = Euclidean distance from worldline
r_{throat} = radius of traversable wormhole throat
ρ = mass density
ρ_{CE} = Casimir Effect energy density
ρ_E = energy density
$\rho_{\text{E-sqvac}}$ = energy density of squeezed electromagnetic vacuum
S^n = n-dimensional sphere
σ = inverse of warp bubble wall thickness
Σ = flat 3-dimensional manifold
T^n = n-dimensional torus
$T_{\mu\nu}$ = vacuum stress-energy tensor
t = time or coordinate time
ϑ = York extrinsic time
θ = spherical polar angle
U = energy density in dimensionless (geometrodynamic) units
$v_{\text{sh}}(t)$ = speed of spaceship motion along worldline (or warp bubble speed)
v_{warp} = dimensionless warp bubble speed
x = space-axis direction or coordinate position
x_{sh} = x-axis coordinate position of moving spaceship frame
ξ = squeezed state amplitude
y = space-axis direction or coordinate position
z = space-axis direction or coordinate position, or laser beam propagation direction
ζ = zeta-function

I. Introduction

It was nearly two decades ago when science fiction media (TV, film, and novels) began to adopt traversable wormholes, and more recently "stargates," for interstellar travel schemes that allowed their heroes to travel throughout our galaxy. In 1985 physicists M. Morris and K. Thorne at CalTech discovered the principle of traversable wormholes right out of Einstein's General Theory of Relativity (GTR, published in 1915). Morris and Thorne[1] and Morris, Thorne and Yurtsever[2] did this as an academic exercise, and in the form of problems for a physics final exam, at the request of Carl Sagan who had then completed the draft of his novel *Contact*. This little exercise ended up becoming one of the greatest cottage industries in general relativity research – the study of traversable wormholes and time machines. Wormholes are hyperspace tunnels through spacetime connecting together either remote regions within our universe or two different universes; they even connect together different dimensions and different times. Space travelers would enter one side of the tunnel and exit out the other, passing through the throat along the way. The travelers would move at $\leq c$ (c is the speed of light) through the wormhole and therefore not violate Special Relativity, but external observers would view the travelers as having traversed multi-light year distances through space at faster-than-light (FTL) speed. A "stargate" was shown to be a very simple special class of traversable wormhole solutions to the Einstein GTR field equations.[3,4]

This development was later followed by M. Alcubierre's discovery in 1994 of the "warp drive" spacetime metric, which was another solution to Einstein's GTR field equations.[5] Alcubierre derived a metric motivated by cosmological inflation that would allow arbitrarily short travel times between two distant points in space. The behavior of the warp drive metric provides for the simultaneous expansion of space behind the spacecraft and a corresponding contraction of space in front of the spacecraft. The warp drive spacecraft would appear to be "surfing on a wave" of spacetime geometry. A spacecraft can be made to exhibit an arbitrarily large apparent FTL speed ($\gg c$) as viewed by external observers, but its moving local rest frame never travels outside of its local comoving light cone and thus does not violate Special Relativity. A cottage industry of warp drive spacetimes research ensued and papers were published during the next 15 years.

The implementation of FTL interstellar travel via traversable wormholes, warp drives, or other FTL spacetime modification schemes generally requires the engineering of spacetime into very specialized local geometries. The analysis of these via Einstein's GTR field equations plus the resultant equations of state demonstrate that such geometries require the use of "exotic" matter in order to induce the requisite FTL spacetime modification. Exotic matter is generally defined by GTR physics to be matter that possesses (renormalized) negative energy density (sometimes negative stress-tension = positive outward pressure, a.k.a. gravitational repulsion or antigravity), and this is a very misunderstood and misapplied term by the non-GTR community. We clear up this misconception by defining what negative energy is, where it can be found in nature, and we also review the experimental concepts that have been proposed to generate negative energy in the laboratory. Also, it has been claimed that FTL spacetimes are not plausible because exotic matter violates the general relativistic energy conditions. However, it has been shown that this is a spurious issue. The identification, magnitude, and production of exotic matter is seen to be a key technical challenge, however. FTL spacetimes also possess features that challenge the notions of causality and there are alleged constraints placed upon them by quantum effects. These issues are reviewed and summarized, and an assessment on the present state of their resolution is provided.

II. The General Relativistic Definition of Exotic Matter and the Energy Conditions

In classical physics the energy density of all observed forms of matter (fields) is non-negative. What is exotic about the matter that must be used to generate FTL spacetimes is that it must have negative energy density and/or negative flux.[6] The energy density is "negative" in the sense that the configuration of matter fields we must deploy to generate and thread a traversable wormhole throat or a warp drive bubble must have an energy density, ρ_E ($= \rho c^2$), that is less than or equal to its pressures, p_i.[1,3] In many cases, these equations of state are also known to possess an energy density that is algebraically negative, i.e., the energy density and flux are less than zero. It is on the basis of these conditions that we call this material property exotic. The condition for ordinary, classical (non-exotic) forms of matter that we are all familiar with in nature is that $\rho_E > p_i$ and/or $\rho_E \geq 0$. These conditions represent two examples of what are variously called the "standard" energy conditions: Weak Energy Condition (WEC: $\rho_E \geq 0$, $\rho_E + p_i \geq 0$), Null Energy Condition (NEC: $\rho_E + p_i \geq 0$), Dominant Energy Condition (DEC), and Strong Energy Condition (SEC). These energy conditions forbid negative energy density between material objects to occur in nature, but they are mere hypotheses. Hawking and Ellis formulated the energy conditions in order to establish a series of mathematical hypotheses governing the behavior of collapsed-matter singularities in their study of cosmology and black hole physics.[7] More specifically, classical general relativity allows one to prove lots of general theorems about the behavior of matter in gravitational fields. Some of the most significant of these general theorems are: focusing theorems (gravitational lensing); singularity theorems (gravity dominated collapse leading to black holes possessing a singularity); positive mass theorem (positive gravitational mass); and topological censorship (you cannot build a wormhole). Each of these requires the existence of "reasonable" types of matter for their formulation, i.e., matter that satisfies the energy conditions.

The bad news is that real physical matter is not "reasonable" because the energy conditions are in general violated by semiclassical quantum effects (occurring at order $\hbar$).[3] More specifically, quantum effects generically violate the average NEC (ANEC). Furthermore, it was discovered in 1965 that quantum field theory has the remarkable property of allowing states of matter containing local regions of negative energy density or negative fluxes.[8] This violates the WEC which postulates that the local energy density is non-negative for all observers. And there are also general theorems of differential geometry that guarantee that there must be a violation of one, some, or all of the energy conditions (meaning exotic matter is present) for all FTL spacetimes. With respect to creating FTL spacetimes, "negative energy" has the unfortunate reputation of alarming physicists. This is unfounded since all the energy condition hypotheses have been experimentally tested in the laboratory and experimentally shown to be false – 25 years before their formulation.[9]

Further investigation into this technical issue showed that violations of the energy conditions are widespread for all forms of both "reasonable" classical and quantum matter.[10-14] Furthermore, M. Visser showed that all (generic) spacetime geometries violate all the energy conditions.[3] So the condition that $\rho_E > p_i$ and/or $\rho_E \geq 0$ must be obeyed by all forms of matter in nature is completely spurious. Violating the energy conditions commits no offense against nature. Negative energy has been produced in the laboratory and this will be discussed in the following sections.

A. Examples of Exotic or "Negative" Energy Found in Nature

The exotic (energy condition-violating) fields that are known to occur in nature are:

1. Static radial electric or magnetic fields. These are borderline exotic, if their tension were infinitesimally larger, for a given energy density.[7,15]
2. Squeezed quantum vacuum states: electromagnetic and other non-Maxwellian quantum fields.[1,16]
3. Gravitationally squeezed vacuum electromagnetic zero-point fluctuations.[17]
4. Casimir Effect, i.e., the Casimir vacuum in flat or curved space.[18-24]
5. Other quantum fields/states/effects. In general, the local energy density in quantum field theory can be negative due to quantum coherence effects.[8] Other examples that have been studied are Dirac field states: the superposition of two single particle electron states and the superposition of two multi-electron-positron states.[25,26] In the former (latter), the energy densities can be negative when two single (multi-) particle states have the same number of electrons (electrons and positrons) or when one state has one more electron (electron-positron pair) than the other.

In addition, cosmological inflation, cosmological particle production, the conformal anomaly, and gravitational vacuum polarization also violate the energy conditions. Since the laws of quantum field theory place no strong restrictions on negative energies and fluxes, then it might be possible to produce exotic phenomena such as FTL travel,[5,27,28] traversable wormholes,[1-3] violations of the second law of thermodynamics,[29,30] and time machines.[2,3,31] There are several other exotic phenomena made possible by the effects of negative energy, but they lie outside the scope of the present study. We review items 1 thru 4 in the following sections and examine their applicability and technical maturity. Dirac field states are currently under study by the author and will therefore be reviewed in a future report.

B. Generating Negative Energy in the Lab

1. Static Radial Electric & Magnetic Fields

It is beyond the scope of this study to include all the technical configurations by which one can generate static radial electric or magnetic fields. Suffice it to say that ultrahigh-intensity tabletop lasers have been used to generate extreme electric and magnetic field strengths in the lab. Ultrahigh-intensity lasers use the chirped-pulse amplification (CPA) technique to boost the total output beam power. All laser systems simply repackage energy as a coherent package of optical power, but CPA lasers repackage the laser pulse itself during the amplification process. In typical high-power short-pulse laser systems, it is the peak intensity, not the energy or the fluence, which causes pulse distortion or laser damage. However, the CPA laser dissects a laser pulse according to its frequency components, and reorders it into a time-stretched lower-peak-intensity pulse of the same energy.[32-34] This benign pulse can then be amplified safely to high energy, and then only afterwards reconstituted as a very short pulse of enormous peak power – a pulse which could never itself have passed safely through the laser system. Made more tractable in this way, the pulse can be amplified to substantial energies (with orders of magnitude greater peak power) without encountering intensity-related problems.

The extreme output beam power, fields and physical conditions that have been achieved by ultrahigh-intensity tabletop lasers are:[34]

a. Power Intensity $\approx 10^{15}$ to 10^{26} W/cm^2 (10^{30} W/cm^2 using SLAC as a booster)
b. Peak Power Pulse $\leq 10^3$ fs
c. E-fields $\approx 10^{14}$ to 10^{18} V/m [note: the critical QED (vacuum breakdown) field intensity is $E_c = 2m_e^2c^3/\hbar e \approx 10^{18}$ V/m (m_e = electron mass, e = electron charge, $\hbar$ = Planck's reduced constant)]
d. B-fields $\approx$ *several* $\times 10^6$ T [note: the critical QED (vacuum breakdown) field intensity is $B_c = E_c/c \approx 10^{10}$ T]
e. Ponderomotive Acceleration of Electrons $\approx 10^{17}$ to 10^{30} $g_\oplus$
f. Light Pressure $\approx 10^9$ to 10^{15} bars

g. Plasma Temperatures > 10^{10} K

The vigilant reader might complain that the electric and magnetic fields generated by ultrahigh-intensity lasers are not static. But in fact, these fields are static over the duration of the pulse-width while at peak intensity. We find from the above data that ultrahigh-intensity lasers can generate an electric field energy density of ~ 10^{16} to 10^{28} J/m^3 and a magnetic field energy density of ~ 10^{19} J/m^3. These energy densities are about the right order of magnitude to explore generating kilometer-to-AU sized wormholes. But that would be difficult to engineer on Earth. However, these energy densities are well above what would be required to explore the generation of micro-wormholes in the lab.

2. Squeezed Quantum Vacuum

Substantial theoretical and experimental work has shown that in many quantum systems the limits to measurement precision imposed by the quantum vacuum zero-point fluctuations (ZPF) can be breached by decreasing the noise in one observable (or measurable quantity) at the expense of increasing the noise in the conjugate observable; at the same time the variations in the first observable, say the energy, are reduced below the ZPF such that the energy becomes "negative." "Squeezing" is thus the control of quantum fluctuations and corresponding uncertainties, whereby one can squeeze the variance of one (physically important) observable quantity provided the variance in the (physically unimportant) conjugate variable is stretched/increased. The squeezed quantity possesses an unusually low variance, meaning less variance than would be expected on the basis of the equipartition theorem. One can in principle exploit quantum squeezing to extract energy from one place in the ordinary vacuum at the expense of accumulating excess energy elsewhere.[1]

The squeezed state of the electromagnetic field is a primary example of a quantum field that has negative energy density and negative energy flux. Such a state became a physical reality in the laboratory as a result of the nonlinear-optics technique of "squeezing," i.e., of moving some of the quantum-fluctuations of laser light out of the $\cos\omega(t - z/c)$ part of the beam and into the $\sin\omega(t - z/c)$ part.[16,35-39] The observable that gets squeezed will have its fluctuations reduced below the vacuum ZPF. The act of squeezing transforms the phase space circular noise profile characteristic of the vacuum into an ellipse, whose semimajor and semiminor axes are given by unequal quadrature uncertainties (of the quantized electromagnetic field harmonic oscillator operators). This applies to coherent states in general, and the usual vacuum is also a coherent state with eigenvalue zero. As this ellipse rotates about the origin with angular frequency ω, these unequal quadrature uncertainties manifest themselves in the electromagnetic field oscillator energy by periodic occurrences, which are separated by one quarter cycle, of both smaller and larger fluctuations compared to the unsqueezed vacuum.

References 1 and 40 point out that if one squeezes the vacuum, i.e., if one puts vacuum rather than laser light into the input port of a squeezing device, then one gets at the output an electromagnetic field with weaker fluctuations and thus less energy density than the vacuum at locations where $\cos^2\omega(t - z/c) \cong 1$ and $\sin^2\omega(t - z/c) << 1$; but with greater fluctuations and thus greater energy density than the vacuum at locations where $\cos^2\omega(t - z/c) << 1$ and $\sin^2\omega(t - z/c) \cong 1$. Since the vacuum is defined to have vanishing energy density, any region with less energy density than the vacuum actually has a negative (renormalized) expectation value for the energy density. Therefore, a squeezed vacuum state consists of a traveling electromagnetic wave that oscillates back and forth between negative energy density and positive energy density, but has positive time-averaged energy density.

For the squeezed electromagnetic vacuum state, the energy density, $\rho_{E\text{-sqvac}}$, is given by:[41]

$$\rho_{E-sqvac} = \left(2\hbar\omega \big/ L^3\right) \sinh\xi \left[\sinh\xi + \cosh\xi \cos\left(2\omega(t - z/c) + \delta\right)\right], \tag{1}$$

where L^3 is the volume of a large box with sides of length L (i.e., we put the quantum field in a box with periodic boundary conditions), ξ is the squeezed state amplitude (giving a measure of the mean photon number in a squeezed state), and δ is the phase of squeezing. Equation (1) shows that the energy density falls below zero once every cycle when the condition $\cosh\xi > \sinh\xi$ is met. It turns out that this is always true for every nonzero value of ξ, so the energy density becomes negative at some point in the cycle for a general squeezed vacuum state. On another note, when a quantum state is close to a squeezed vacuum state, there will almost always be some negative energy densities present.

Negative energy can be generated by an array of ultrahigh intensity lasers using an ultrafast rotating mirror system.[42] In this scheme a laser beam is passed through an optical cavity resonator made of a lithium niobate ($LiNbO_3$) crystal that is shaped like a cylinder with rounded silvered ends to reflect light. The resonator will act to produce a secondary lower frequency light beam in which the pattern of photons is rearranged into pairs. The squeezed light beam emerging from the resonator will contain pulses of negative energy interspersed with pulses of positive energy.

In this example both the negative and positive energy pulses are of ~ 10^{-15} second duration. We could in principle arrange a set of rapidly rotating mirrors to separate the positive and negative energy pulses from each other. The light beam would be set to strike each mirror surface at a very shallow angle while the rotation would ensure that the negative energy pulses would be reflected at a slightly different angle from the positive energy pulses. A small spatial separation of the two different energy pulses would occur at some distance from the rotating mirror. Another system of mirrors would be needed to redirect the negative energy pulses to an isolated location and concentrate them there.

The rotating mirror system can actually be implemented via non-mechanical means. A chamber of sodium gas is placed within the squeezing cavity, and a laser beam is directed through the gas. The beam is reflected back on itself by a mirror to form a standing wave within the sodium chamber. This wave causes rapid variations in the optical properties of the sodium thus causing rapid variations in the squeezed light so that we can induce rapid reflections of pulses by careful design.[36]

Another way to generate negative energy via squeezed light would be to manufacture extremely reliable light pulses containing precisely one, two, three, etc., photons apiece and combine them together to create squeezed states to order. Reference 42 points out that superimposing many such states could theoretically produce bursts of intense negative energy. Photonic crystal research has already demonstrated the feasibility of using photonic crystal waveguides (mixing together the classical and quantum properties of optical materials) to engineer light sources that produce beams containing precisely one, two, three, etc., photons. For example, researchers at Melbourne University used a microwave oven to fuse a tiny diamond, just 1/1000th of a millimeter long, onto an optical fiber, which could be used to create a single photon beam of light.[43,44] The combining of different beams containing different (finite integer) numbers of photons is already state-of-the-art practice via numerous optical beam combining methods that can readily be extended to our application.

Finally, Ref. 45 experimentally demonstrated the very first simple, scalable squeezed vacuum source in the laboratory that consisted of a continuous-wave diode laser and an atomic rubidium vapor cell. The experimental tools we need to begin exploring the generation of negative energy for the purpose of creating a FTL spacetime are just now becoming available.

3. Gravitationally Squeezed Electromagnetic ZPF

A natural source of negative energy comes from the effect that gravitational fields (of astronomical bodies) in space have upon the surrounding vacuum. For example, the gravitational field of the Earth produces a zone of negative energy around it by dragging some of the virtual particle pairs (a.k.a. vacuum ZPF) downward. This concept was initially developed in the 1970s as a byproduct of studies on quantum field theory in curved space.[21] However, D. Hochberg and T. W. Kephart[17] derived an important application of this concept to the problem of creating and stabilizing traversable wormholes, and their work was later corrected and extended by the author.[46] They proved that one can utilize the negative energy densities, which arise from distortion of the vacuum electromagnetic ZPF due to the interaction with a prescribed gravitational background, for providing a violation of the energy conditions. The squeezed quantum states of quantum optics provide a natural form of matter having negative energy density.

The analysis, via quantum optics, showed that gravitation itself provides the mechanism for generating the squeezed vacuum states needed to support stable traversable wormholes. The production of negative energy densities via a squeezed vacuum is a necessary and unavoidable consequence of the interaction or coupling between ordinary matter and gravity, and this defines what is meant by gravitationally squeezed vacuum states. The magnitude of the gravitational squeezing of the vacuum can be estimated from the quantum optics squeezing condition for given transverse momentum and (equivalent) energy eigenvalues, j, of two electromagnetic ZPF field modes, such that this condition is subject to $j \to 0$, where[17,46]

$$\begin{aligned} j &\equiv \frac{4\pi c^2}{\lambda g_\oplus}\left(\frac{r}{R_\oplus}\right)^2 \frac{M_\oplus}{M} , \\ &= \frac{8\pi r_S}{\lambda} \end{aligned} \tag{2}$$

λ is the ZPF mode wavelength, $g_\oplus$ is the acceleration of gravity at the Earth's surface, r is the radial distance from the center of the astronomical body in question, $R_\oplus$ is the radius of the Earth, $M_\oplus$ is the mass of the Earth, M is the mass of the astronomical body, and r_S is the Schwarzschild radius of the astronomical body. (Note: r_S is only a convenient radial distance parameter for any massive body under examination and so there is no black hole collapse involved in this analysis. The Schwarzschild radius is the critical radius, according to GTR, at which a spherically symmetric massive body becomes a black hole, i.e., at which light is unable to escape from the body's surface. We can actually choose any radial distance from the body in question to perform this analysis, but using r_S makes the equation simpler in form.) The squeezing condition plus Eq. (2) simply states that substantial gravitational squeezing of the vacuum occurs for those ZPF field modes with $\lambda \geq 8\pi r_S$ of the mass in question (whose gravitational field is squeezing the vacuum).

The general result of the gravitational squeezing effect is that as the gravitational field strength increases, the negative energy zone (surrounding the mass) also increases in strength. Table 1 shows when gravitational squeezing becomes important for sample masses. The table shows that in the case of the Earth, Jupiter and the Sun, this squeeze effect is extremely feeble because only ZPF mode wavelengths above 0.2 m to 78 km are affected. For a solar mass black hole (radius of 2.95 km), the effect is still feeble because only ZPF mode wavelengths above 78 km are affected. But note from the table that Planck mass objects will have an enormously strong negative energy zone surrounding them because all ZPF mode wavelengths above 8.50×10^{-34} meters will be squeezed, in other words, all wavelengths of interest for vacuum fluctuations. Protons will have the strongest negative energy zone in comparison because the squeezing effect includes all ZPF mode wavelengths above 6.50×10^{-53} meters. Furthermore, a body smaller than a nuclear diameter ($\approx 10^{-16}$ m) and containing the mass of a mountain ($\approx 10^{11}$ kg) has a fairly strong negative energy zone because all ZPF mode wavelengths above 10^{-15} meters will be squeezed.

Table 1. Substantial Gravitational Squeezing Occurs for Vacuum ZPF When $\lambda \geq 8\pi r_S$.[46]

Mass of body (kg)	Schwarzschild radius of body, r_S (m)	ZPF mode wavelength, λ (m)
Sun = 2.00×10^{30}	2.95×10^{3}	$\geq 78.0 \times 10^{3}$
Jupiter = 1.90×10^{27}	2.82	≥ 74
Earth = 5.98×10^{24}	8.87×10^{-3}	≥ 0.23
Typical mountain $\approx 10^{11}$	$\approx 10^{-16}$	$\geq 10^{-15}$
Planck mass = 2.18×10^{-8}	3.23×10^{-35}	$\geq 8.50 \times 10^{-34}$
Proton = 1.67×10^{-27}	2.48×10^{-54}	$\geq 6.50 \times 10^{-53}$

We are presently unaware of any way to artificially generate gravitational squeezing of the vacuum in the laboratory. This will be left for future investigation. However, it is predicted to occur in the vicinity of astronomical matter. Naturally occurring traversable wormholes in the vicinity of astronomical matter would therefore become possible.

4. Vacuum Field Stress: Negative Energy from the Casimir Effect

The Casimir Effect is by far the easiest and most well known way to generate negative energy in the lab. The Casimir Effect that is familiar to most people is the force that is associated with the electromagnetic quantum vacuum.[47] This is an attractive force that must exist between any two neutral (uncharged), parallel, flat, conducting surfaces (e.g., metallic plates) in a vacuum. This force has been well measured and it can be attributed to a minute imbalance in the electromagnetic zero-point energy density inside the cavity between the conducting surfaces versus the region outside of the cavity.[48-50]

It turns out that there are many different types of Casimir Effects found in quantum field theory.[18-20,24,51] For example, if one introduces a single infinite plane conductor into the Minkowski (flat spacetime) vacuum by bringing it adiabatically from infinity so that whatever quantum fields are present suffer no excitation but remain in their ground states, then the vacuum (electromagnetic) stresses induced by the presence of the infinite plane conductor produces a Casimir Effect. This result holds equally well when two parallel plane conductors (with separation distance d) are present, which gives rise to the familiar Casimir Effect inside a cavity. Note that in both cases, the spacetime manifold is made incomplete by the introduction of the plane conductor boundary condition(s). The vacuum region put under stress by the presence of the plane conductor(s) is called the Casimir vacuum. The generic expression for the energy density of the Casimir Effect is $\rho_{CE} = -A(\hbar c)d^{-4}$, where $A = \zeta(D)/8\pi^2$ in spacetimes of arbitrary dimension D.[18-20] The appearance of the zeta-function $\zeta(D)$ is characteristic of expressions for vacuum stress tensors, $T_{\mu\nu}$. In our familiar 4-dimensional spacetime ($D = 4$) we have that $A = \pi^2/720$. To calculate $T_{\mu\nu}$ for a given quantum field is to calculate its associated Casimir Effect.

Analogs of the Casimir Effect also exist for fields other than the electromagnetic field. When considering the vacuum state of other fields, one must consider boundary conditions that are analogous to the perfect-conductor boundary conditions for the electromagnetic field at the surfaces of the plates.[18-20,24] Other fields are not electromagnetic in nature, that is to say they are non-Maxwellian, and so the perfect-conductor boundary conditions do not apply to the other fields. It turns out that complete manifolds exhibit the Casimir Effect for any non-Maxwellian fields. In order to define boundary conditions for other fields we replace the conductor boundary conditions (plates) and Minkowski spacetime by a manifold of the form $\Re \times \Sigma$ (i.e., a product space), where $\Re$ is the real line defining the time dimension for this particular product space and Σ is a flat 3-dimensional manifold (3-manifold) having any one of the following topologies: $\Re^2 \times S^1$, $\Re \times T^2$, T^3, $\Re \times K^2$, etc., $\Re$ being the real line that defines any linear space dimension (e.g., $\Re$ = line, $\Re^2$ = 2-dimensional plane, etc.), T^n being the n-torus, K^2 the 2-dimensional Klein bottle, S^1 the circle, etc.

The case $\Sigma = \Re^2 \times S^1$ has the closest resemblance to the electromagnetic Casimir Effect, the difference being that instead of imposing conductor boundary conditions, one imposes periodic boundary conditions on some of the space coordinates in the 3-manifold. When imposing this topological constraint on the field theoretic calculation of the Casimir Effect (for linear massless fields), one finds that the generic expression for the energy density of the Casimir Effect is $\rho_{CE} = -A(\hbar c)d^{-4}$, where $A = \pm d_f(\pi^2/90)$ and d_f is the number of degrees of freedom (e.g., helicity states) per spatial point and where the plus sign holds for boson fields (giving a negative energy density) and the minus sign for fermion fields (giving a positive energy density).

If one were to admit spin structure in the manifolds described above and the field is spinorial, then there is another important subtlety that must be taken into account when evaluating the vacuum $T_{\mu\nu}$. However, this introduces an additional complexity involving the relationship between the spin structure and the global structure (i.e., the configuration space or fibre bundle) of the field in question whereby the topology not only of the base manifold but of the fibre bundle itself has an effect on $T_{\mu\nu}$. A consideration of the various quantum field analogs of the Casimir Effect for generating FTL spacetimes will be left for future investigation.

As a final note, we point out that the methods used to obtain the vacuum electromagnetic $T_{\mu\nu}$ between parallel plane conductors can also be used when the conductors are not parallel but are joined together along a line of intersection. If the conductors have curved surfaces instead, then one obtains results that are similar to the intersecting conductors case. These geometries have also been evaluated for the case of dielectric media. These particular cases will not be considered further since there are technical subtleties involved that complicate the calculations and application of the different approaches.

4a. Casimir Effect: Moving Mirrors

Negative energy can be created by a single moving reflecting (conducting) surface (a.k.a. a moving mirror). A mirror moving with increasing acceleration generates a flux of negative energy that emanates from its surface and flows out into the space ahead of the mirror.[21,52] This is essentially the simple case of an infinite plane conductor undergoing acceleration perpendicular to its surface. If the acceleration varies with time, the conductor will generally emit or absorb photons (i.e., exchange energy with the vacuum), even though it is neutral. This is an example of the well-known quantum phenomenon of parametric excitation. The parameters of the electromagnetic field oscillators (e.g., their frequency distribution function) change with time owing to the acceleration of the mirror (conductor).[53] However, this effect is known to be exceedingly small, and it is not the most effective way to generate negative energy for our purposes. We will not consider this scheme any further.

4b. Casimir Effect: Negative Energy for Traversable Wormholes

The (electromagnetic) Casimir Effect can be used to create a traversable wormhole. The energy density $\rho_{CE} = -(\pi^2 \hbar c/240)d^{-4}$ J/m^3 within a Casimir cavity is negative and manifests itself by producing a force of attraction between the cavity walls. But cavity dimensions must be made exceedingly small in order to generate a significant amount of negative energy for our purposes. In order to use the Casimir Effect to generate a spherically symmetric traversable wormhole throat of radius r_{throat}, we will need to design a cavity made of perfectly conducting spherically concentric thin plates with a plate separation d (in meters) of:[2]

$$\begin{aligned} d &= \left(\pi^3/30\right)^{1/4} \left(r_{throat}\sqrt{\hbar G/c^3}\right)^{1/2}, \\ &= \left(4.05\times10^{-18}\right)\sqrt{r_{throat}} \end{aligned} \tag{3}$$

where r_{throat} is in meters and G is the universal gravitation constant. To counteract the collapse of the cavity due to the Casimir Force acting between the plates, the plates will have equal electric charges placed upon them to establish adequate Coulomb repulsion. (In a detailed analysis the electrostatic energy required to support the Coulomb repulsion between the plates would be considered separately.) Equation (3) shows that a 1 km radius throat will require a cavity plate separation of 1.28×10^{-16} m (smaller than a nuclear diameter), and the energy density ρ_{EC} generated by this configuration is -4.83×10^{36} J/m^3. In contrast, a wormhole with a throat radius of 1 AU will require a plate separation of 1.57×10^{-12} m (or 35% smaller than the electron's Compton wavelength), and generate -2.16×10^{20} J/m^3 of energy density. There is no technology known today that can engineer a cavity with such miniscule plate separations. In addition, such miniscule plate separations are unrealistic because the Casimir Effect switches over to the non-retarded field behavior ($\sim d^{-3}$) of van der Waals forces when plate separations go below the wavelength ($\approx$ 15 nm) where they are no longer perfectly conducting.[54] We will not consider this scheme further. However, future work will be necessary to elucidate whether the various quantum field analogs of the Casimir Effect can provide a more reasonable technical solution.

III. A Brief Review of FTL Spacetimes

In Sect. I we briefly described the two primary classes of FTL spacetimes found in GTR: traversable wormholes and warp drives. There are no other (credible) classes of FTL spacetimes having to do with GTR, higher dimensional D-Brane theory, or (credible) alternative theories of gravity (e.g., Puthoff's Polarizable-Vacuum General Relativity Theory, Yilmaz Theory, etc.). Investigators have discovered that GTR and all of the (credible) alternative theories of gravity are infested with traversable wormholes, time machines, and some form of a warp drive. The small number of proposed alternative FTL spacetimes are largely based on spurious physics and are thus highly suspect.

How does one study the physics of FTL spacetimes within the framework of GTR? When studying spacetime physics, the normal philosophy is to take the Einstein field equations, add some form of matter, make simplifying assumptions, and then solve to deduce what the geometry of spacetime will be. This is very difficult to do because there are ten nonlinear second-order partial differential equations with four redundancies (arbitrary choice of spacetime coordinates) and four constraints (stress-energy conservation). There is a tremendous body of research that takes exactly this approach, either analytically or numerically. However, this is not the best strategy for understanding FTL spacetimes. The appropriate strategy is to decide beforehand on a definition of the traversable wormhole or warp drive that you desire and decide what the spacetime geometry should look like. Given the desired geometry, use the Einstein field equations to calculate the distribution of matter required to set up this geometry. Then one should ask themselves whether the required distribution of matter is physically reasonable and whether it violates any basic rules of physics, etc. In the following sections we briefly outline the key results for traversable wormholes and warp drive spacetimes.

A. Traversable Wormholes

Traversable wormholes represent a class of exact metric solutions of Einstein's GTR field equations. The solutions are "exact" in the sense that no approximations requiring a plethora of physical assumptions have to be made to derive the appropriate spacetime geometry. To define a stable traversable wormhole one needs to define the desirable physical requirements it is to have in order to achieve the desired FTL travel benefit. The requirements we desire are the following:[1,3]

1. Travel time through the wormhole tunnel or throat should be $\leq$ 1 year as seen by both the travelers and outside static observers.
2. Proper time as measured by travelers should not be dilated by relativistic effects.
3. The gravitational acceleration and tidal-gravity accelerations between different parts of the travelers' body should be $\leq 1\, g_{\oplus}$ when going through the wormhole.
4. Travel speed through the tunnel/throat should be $< c$.
5. Travelers (made of ordinary matter) must not couple strongly to the material that generates the wormhole curvature; the wormhole must be threaded by a vacuum tube through which the travelers can move.
6. There is no event horizon at the wormhole throat.
7. There is no singularity of infinitely collapsed matter residing at the wormhole throat.

These requirements then lead us to define a spherically symmetric Lorentzian spacetime metric (i.e., invariant distance function in spacetime), ds^2, that prescribes the required traversable wormhole geometry:[1,3]

$$ds^2 = -e^{2\phi(r)}c^2dt^2 + [1 - b(r)/r]^{-1}dr^2 + r^2d\Omega^2, \quad (4)$$

where standard spherical coordinates are used (r: $2\pi r$ = circumference; $0 \le \theta \le \pi$; $0 \le \varphi \le 2\pi$), t is time ($-\infty < t < \infty$), c is the speed of light, $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, $\phi(r)$ is the freely specifiable redshift function that defines the proper time lapse through the wormhole throat, and $b(r)$ is the freely specifiable shape function that defines the wormhole throat's spatial (hypersurface) geometry. The throat is spherically shaped. There are several variations of Eq. (4), which define traversable wormholes having different properties. The reader should consult Ref. 3 for further details.

Figure 1 shows two diagrams representing the embedded space (Flamm) representation of Eq. (4), which depicts the geometry of an equatorial ($\theta = \pi/2$) slice through space at a specific moment of time (t = const). The top of Fig. 1 shows the embedding diagram for a traversable wormhole that connects two different universes (i.e., an inter-universe wormhole). The bottom diagram in the figure is an intra-universe wormhole with a throat that connects two distant regions of our own universe. These diagrams serve to aide in visualizing traversable wormhole geometry and are merely a geometrical exaggeration.

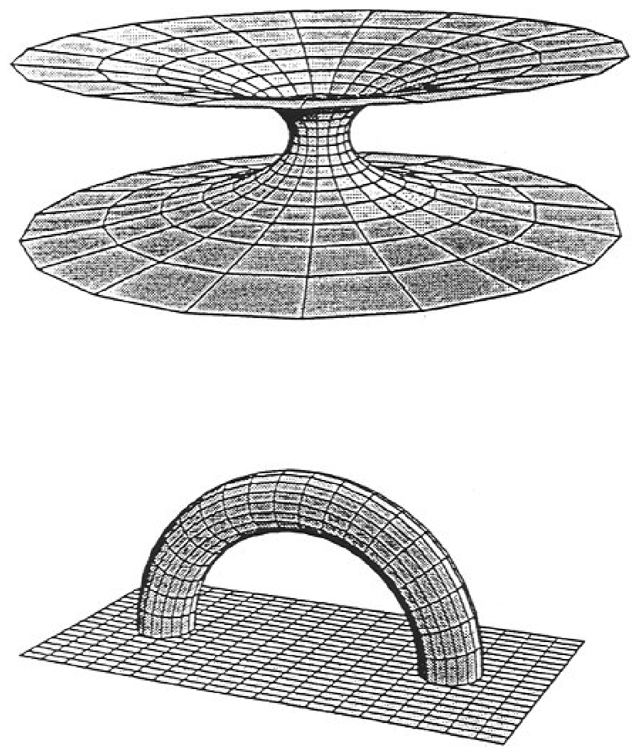

Figure 1. Inter-Universe Wormhole (top) and Intra-Universe Wormhole (bottom).

There was originally one other criterion for defining a traversable wormhole, which was that it must be embedded within the surrounding (asymptotically) flat spacetime. However, Ref. 55 proved that it is only the behavior near the wormhole throat that is critical to understanding the physics, and that a generic throat can be defined without having to make all the symmetry assumptions and without assuming the existence of an asymptotically flat spacetime in which to embed the wormhole. Therefore, one only needs to know the generic features of the geometry near the throat in order to guarantee violations of the NEC for certain open regions near the throat. So we are free to place our wormhole anywhere in spacetime we want to because it is only the geometry and physics near the throat that matters for any analysis. This fact led to the development of a number of different traversable wormhole throat designs that are cubic shaped, polyhedral shaped, flat-face shaped, generic shaped, etc. The reader should consult Ref. 3 for a complete technical review of the various types (and shapes) of traversable wormhole solutions found in GTR.

We know that we need exotic or negative energy to create and thread open a traversable wormhole. So in this regard, we ask what kind of wormhole can one make with less effort. To answer this question we can relate the local wormhole geometry to the global topological invariant of the spacetime via the Gauss-Bonnet Theorem.[56] In the Gauss-Bonnet Theorem the local wormhole geometry is quantified by the energy density, U (in dimensionless geometrodynamic units, $\hbar = G = c = 1$), threading the wormhole throat plus a spatial curvature constant (for the throat). The global topological invariant of spacetime is quantified by the Euler Number, χ_e, which is itself defined in terms of the genus, g, representing the number of handles or throats (or tunnels) a wormhole can be assigned. These two topological quantities are related via $\chi_e = 2(1 - g)$. Therefore, the (static) wormhole Gauss-Bonnet relation is given by $U \le \chi_e/4$ or $U \le (1 - g)/2$.[56] (The case for dynamic traversable wormholes has results that are not too dissimilar from the static case.) This relation will help us to decide if we want to build a traversable wormhole having one throat, or two or more throats, and at what energy cost this will incur.

The following is the result of our analysis for traversable wormholes having:

a) 1-handle/throat (i.e., flat torus or spherical wormhole topology) giving $g = 1$, thus $\chi_e = 0$, and so $U \le 0$.

b) 2-handles/throats giving $g = 2$, thus $\chi_e = -2$, and so $U \leq -1/2$.
c) 3-handles/throats giving $g = 3$, thus $\chi_e = -4$, and so $U \leq -1$; and so on.

It is clear from this that as the number of wormhole handles/throats increases the amount of negative energy required to create the wormhole will grow larger in magnitude. This is an undesirable demand on our putative negative energy generator. It is clear then that item (a) defines the most desirable engineering solution we can hope for: a 1-handle/throat traversable wormhole that will require zero or (arbitrarily) little negative energy to create. The magnitude of energy condition violations and the amount of negative energy required to build a traversable wormhole will be addressed in Sec. IV.

B. Warp Drives

M. Alcubierre[5] derived a spacetime metric motivated by cosmological inflation that would allow arbitrarily short travel times between two distant points in space. The "warp drive" metric uses coordinates (t, x, y, z) and curve (or worldline) $x = x_{sh}(t)$, $y = 0$, $z = 0$, lying in the t-x plane passing through the origin. Note that x_{sh} is the x-axis coordinate position of the moving spaceship (or warp bubble) frame. The metric, ds^2, specifying this spacetime is:[5]

$$ds^2 = -c^2 dt^2 + \left[dx - v_{sh}(t) f(r_{sh}) dt\right]^2 + dy^2 + dz^2 , \tag{5}$$

where $v_{sh}(t) \equiv dx_{sh}(t)/dt$ is the speed associated with the curve (or warp bubble speed), $r_{sh} \equiv [(x - x_{sh}(t))^2 + y^2 + z^2]$ is the Euclidean distance from the curve. The warp bubble shape function $f(r_{sh})$ is any smooth positive function that satisfies $f(0) = 1$ and decreases away from the origin to vanish when $r_{sh} > R$ for some R. The geometry of each spatial slice is flat, and spacetime is flat where $f(r_{sh})$ vanishes but is curved where it does not vanish.

The driving mechanism of Eq. (5) is the York extrinsic time, ϑ. This quantity is defined as:[5]

$$\vartheta = \frac{v_{sh}}{c} \frac{x_{sh}}{r_{sh}} \frac{df}{dr_{sh}} . \tag{6}$$

The ϑ behavior of the warp drive bubble provides for the simultaneous expansion of space behind the spacecraft and a corresponding contraction of space in front of the spacecraft. Figure 2 illustrates the ϑ behavior of the warp drive bubble geometry. Thus the spacecraft is enveloped within a warp bubble and can be made to exhibit an arbitrarily large FTL speed ($v_{sh} \gg c$) as viewed by external coordinate observers. Even though the worldlines inside the warp bubble region are spacelike for all external observers, the moving spaceship (warp bubble) frame itself never travels outside of its local comoving light cone and thus does not violate Special Relativity.

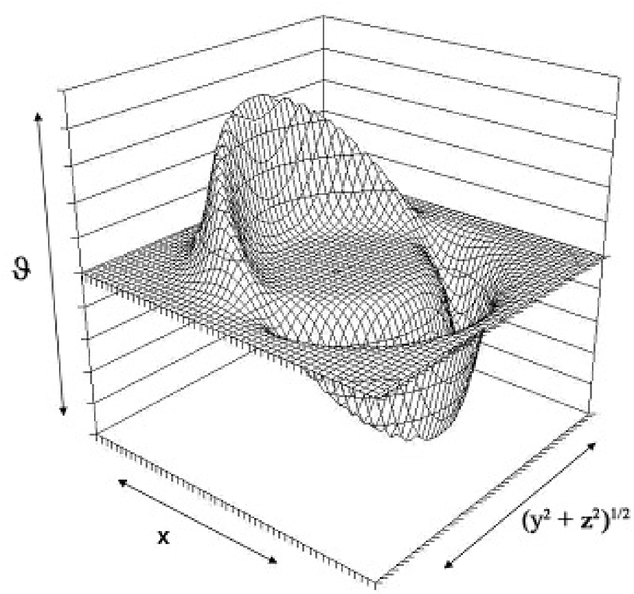

Figure 2. York Extrinsic Time (ϑ) Plot.

Other investigators have designed warp drive metrics similar to Eq. (5) but with some modifications to the geometry and corresponding negative energy requirement.[57-64] Reference 65 gives one example of how Alcubierre's warp drive can be reinterpreted in extra-dimensional D-Brane theory as a spacetime expansion boost (i.e., like a scalar multiplier acting) on the initial spacecraft speed. This mechanism recasts the warp drive energy requirement into the equation of state for dark energy (a.k.a. the cosmological vacuum energy) whereby there is no negative energy density, but only negative pressure, that is required to build the warp drive bubble. Reference 66 is another example whereby Alcubierre's warp drive can be engineered via an effective (broken) supersymmetry Casimir energy in extra-dimensional D-Brane theory. Finally, Puthoff[67] showed that Alcubierre's warp drive is a particular case of a broad, general approach that is called "metric engineering," the details of which provide support for the concept that reduced-time interstellar travel is not fundamentally constrained by physical principles. The magnitude of energy condition violations and the amount of negative energy required to build an Alcubierre-type warp drive will be addressed in the next section.

IV. The Make or Break Issues

A. Negative Energy Requirements and Energy Condition Violations

We know how to make small quantities of negative energy in the lab. But we do not know if it is possible to make large quantities of negative energy. It was pointed out in Sec. II that one, some, or all of the energy conditions must be violated in order to build a FTL spacetime. And we also cautioned that this was not a showstopper because the energy conditions have all been violated by nature or by lab experiment prior to their formulation. However, the reader should be forewarned that there are a number of published claims that energy condition violations can be avoided. These claims are just semantic games whereby investigators universally invoke the following scenario: divide the total stress-energy into weird matter plus normal matter, push all the energy condition violations into the weird matter so that the normal matter does not violate the energy conditions. This is completely spurious physics and should be ignored.

1. Traversable Wormhole Requirements

Traversable wormhole throats violate the NEC (or ANEC). So we ask how big a violation is required. The answer is that we only need to calculate the amount of negative energy that will be needed to generate and hold open a wormhole throat. A simple formula for short-throat wormholes using the thin shell formalism gives this quantity in terms of the equivalent mass (note: the energy density derived from the GTR field equations is too complex to use):[3]

$$\begin{aligned} M_{wh} &= -\frac{r_{throat}\,c^2}{G} \\ &= -(1.35\times 10^{27}\,kg)\,\frac{r_{throat}}{1\,meter}, \\ &= -(0.71 M_J)\,\frac{r_{throat}}{1\,meter} \end{aligned} \tag{7}$$

where M_{wh} is the (equivalent) mass required to build the wormhole, r_{throat} is a suitable measure of the linear dimension (radius) of the throat, and M_J is the mass of the planet Jupiter. One can also obtain the required energy, E_{wh}, by multiplying both sides of Eq. (7) by c^2. Equation (7) shows that a mass of $-0.709M_J$ will be required to build a wormhole 1 meter in size. As the wormhole size increases, the mass requirement grows negative-large. Table 2 presents a tabulation of the required negative (equivalent) mass as a function of sample wormhole throat sizes. After being alarmed by the magnitude of this, one should note that M_{wh} is not the total mass of the wormhole as seen by remote observers. The non-linearity of the Einstein field equations dictates that the total mass is zero (actually, the total net mass being positive, negative or zero in the Newtonian approximation depending on the details of the negative energy configuration constituting the wormhole system). Finally, Ref. 68 demonstrated the existence of spacetime geometries containing traversable wormholes that are supported by arbitrarily small quantities of negative energy, and this was proved to be a general result. Therefore, fortunately, we can safely ignore Eq. (7) and Table 2. We will expand on this further in Section IVB.

Table 2. Negative Mass Required for Traversable Wormhole

Wormhole throat radius, r_{throat} (m)	Required mass, M_{wh}
1000	$-709.9\ M_J$
100	$-71\ M_J$
10	$-7.1\ M_J$
1	$-0.71\ M_J$
0.1	$-22.6\ M_\oplus$
0.01	$-2.3\ M_\oplus$

$M_J = 1.90 \times 10^{27}$ kg, $M_\oplus = 5.98 \times 10^{24}$ kg

2. Warp Drive Requirements

Warp drives violate the WEC and NEC (also the DEC and SEC). The amount of negative energy required to create a warp bubble gives a measure of the WEC/NEC violations. Because the energy *density* for the Alcubierre[5] (and Natário[69]) warp drive that is derived from the GTR field equations is too complex to use, we instead use a more simple formula to express the net energy required, E_{warp}, to build a warp bubble around a spaceship:[70]

$$
\begin{aligned}
E_{warp} &= -\frac{v_{warp}^2 c^4 R^2 \sigma}{G} \\
&= -\left(1.21\times10^{44}\right) v_{warp}^2 R^2 \sigma
\end{aligned}
\quad , \qquad (8)
$$

where v_{warp} [v_{warp}: $(0,\infty)$] is the dimensionless speed of the warp bubble, R (> 0) is the radius of the warp bubble, and σ (> 0) is proportional to the inverse of the warp bubble wall thickness Δ (i.e., σ ~ 1/Δ). One can also obtain the equivalent mass, M_{warp}, by dividing both sides of Eq. (8) by c^2. Equation (8) characterizes the amount of negative energy that one needs to localize in the walls of the warp bubble. Table 3 presents a tabulation of the required negative energy as a function of the "warp factor," v_{warp}. One can compare the values of E_{warp} in the table with the (positive) mass-energy contained in the Sun (1.79×10^{47} J). The consequence of Eq. (8) and Table 3 is that if one wants to travel at hyperlight speeds, then the warp bubble energy requirement will be an enormous negative number. And this remains true even if one engineers an (arbitrarily low) sublight speed warp bubble. Engineering a warp drive bubble is quite daunting given these results.

Table 3. Negative Energy Required for Warp Bubble

Warp Factor, v_{warp}	Required E_{warp} (J)
10^{-5} (= 3 km/sec)	-3.03×10^{40}
10^{-4} (= 30 km/sec)	-3.03×10^{42}
0.01 (= 3,000 km/sec)	-3.03×10^{46}
0.5 (= 150,000 km/sec)	-7.59×10^{49}
1 (= speed of light)	-3.03×10^{50}
2 (= 600,000 km/sec)	-1.21×10^{51}
10 (= 3.0×10^6 km/sec)	-3.03×10^{52}
100 (= 3.0×10^7 km/sec)	-3.03×10^{54}

Assume: $R = 50$ m, $\sigma = 10^3$ m^{-1}

One further complication arises from the serious flaw that was discovered in the Alcubierre and Natário warp drives by F. Lobo and M. Visser.[70] They point out that in the original version of the warp field, the point at the center of the warp bubble moves on a geodesic and is massless. The spaceship is always treated as a test particle in Alcubierre's warp drive and all other incarnations of it. Lobo and Visser corrected this flaw by constructing a more realistic model of the warp drive spacetime by applying linearized gravity to the weak-field warp drive case and testing the energy conditions to first and second orders of v_{warp}. The fundamental basis of their model is that it specifically includes a finite mass spaceship that interacts with the warp bubble. Their results verified that all warp drive spacetimes violate the energy conditions and will continue to do so for arbitrarily low warp bubble speed. They also found that the energy condition violations in this class of spacetimes is generic to the form of the geometry under consideration and is not a side-effect of the superluminal properties. Based on these facts plus Eq. (8) and Table 3, it appears that for all conceivable laboratory experiments in which negative energy can be created in very small amounts, the warp bubble speed will be absurdly low. It therefore appears unlikely that warp drives will ever prove to be technologically practical unless new warp bubble geometries or ways to generate astronomical amounts of negative energy are found.

B. Quantum Inequalities and Spatial Distributions of Negative Energy

The Quantum Inequalities (QI) conjecture is an ad hoc extension of the Heisenberg Uncertainty Principle to curved spacetimes. A small cottage industry of research has formed around this one topic alone. The literature is too numerous to cite here but the reader should consult Refs. 6 and 41 for detailed information. The QI conjecture relates (via model dependent time integrals of the energy density along geodesics) the energy density of a free quantum field and the time during which this energy density is observed. This conjecture was devised as an attempt to quantify the amount of negative energy or energy condition violations required to build a FTL spacetime. Investigators have invoked the QI to rule out many of the macroscopic wormhole and warp drive spacetimes. When generating negative energy the QI postulate that: a) the longer the pulse of negative energy lasts, the weaker it must be; b) a pulse of positive energy must follow and the magnitude of the positive pulse must exceed that of the initial negative pulse; and c) the longer the time interval between the two pulses, the larger the positive pulse must be. This actually sounds quite reasonable on energy conservation grounds until one discovers that the Casimir Effect and its non-Maxwellian quantum field analogs violate all three conditions. There are also a number of squeezed vacuum sources and Dirac field states that manifestly violate all three conditions. Cosmological inflation, cosmological particle production, the conformal anomaly, and gravitational vacuum polarization also violate the QI. M. Visser[71] also points out that observational data indicate that large amounts of "exotic matter" are required to exist in the universe in order to account for the observed cosmological evolution parameters. The QI have also not been verified by laboratory experiments. The assumptions used to derive the QI and the efficacy of their derivation for various cases has been called into question by numerous investigators. S. Krasnikov[72] constructed an explicit counterexample for generalized FTL spacetimes showing that the relevant QI breaks down even in the simplest FTL cases. Therefore, the QI conjecture is flawed and we will not consider it any further.

It turns out that S. Kar, N. Dadhich and M. Visser[61,68,70,73] developed a superior way to properly quantify the amount of negative energy or energy condition violations required to build a FTL spacetime. They propose a quantifier in terms of a spatial volume integral, which amounts to calculating the following definite integrals:[68,70,73]

$$\int \rho_E \, dV ; \quad \int \left(\rho_E + p_i \right) dV \tag{9}$$

with an appropriate choice of the integration measure dV [$4\pi r^2 dr$ or $(g_{\mathrm{metric}})^{1/2}\, dr\, d\theta\, d\varphi$]. The amount of energy condition violation is defined as the extent to which Eq. (9) can become negative. The value of Eq. (9) provides information about the total amount of energy condition violating matter that must exist for any given FTL spacetime under examination. It was further shown that Eq. (9) can be adjusted to become vanishingly small by appropriate choice of parameters; therefore, examples can be constructed whereby the energy condition violation can be made arbitrarily small. But the violation cannot be made to vanish entirely.

Equation (8) is also a consequence of Eq. (9). Coupling of the finite spaceship mass with the warp bubble leads to the (quite reasonable) condition that the net total energy stored in the warp field be less than the total mass-energy of the spaceship itself, which places a strong constraint upon the (dimensionless) speed of the warp bubble:[70]

$$\begin{aligned} v_{warp} &\le \left[\frac{G}{c^2} \left(\frac{M_{ship}}{R_{ship}} \frac{R_{ship}\,\Delta}{R^2} \right) \right]^{1/2} \\ &\le \left[\left(7.41 \times 10^{-28} \right) \left(\frac{M_{ship}\,\Delta}{R^2} \right) \right]^{1/2} \end{aligned} , \tag{10}$$

where M_{ship} and R_{ship} are the mass and size of the spaceship, respectively. (One can multiply Eq. (10) by c to convert to MKS units of speed.) Equation (10) indicates that for any reasonable values of the engineering parameters inside the brackets, v_{warp} will be absurdly low. This result is due to the intrinsic nonlinearity of the Einstein field equations.

Equation (9) also gives the result that traversable wormholes require arbitrarily small amounts of negative energy to build (whereby Eq. (7) serves only as a gross upper limit) such that within a wormhole spacetime we must have that:[68]

$$\rho_E = 0 ; \quad \int_C p_r \, dV \to 0 , \tag{11}$$

where p_{r} is the outward radial pressure required to hold the wormhole throat open. We should point out that the Gauss-Bonnet Theorem (Sect. IIIA) predicted this result beforehand. Equation (11) is a result that is also due to the intrinsic nonlinearity of the Einstein field equations. This nonlinearity also impacts the coupling of a finite spaceship mass with each side of a wormhole's throat (or the mouth on each side of the throat) leading to a specialized mass conservation law for the combined system of spacecraft and wormhole: when finite mass spaceships traverse a wormhole they alter the (equivalent) mass of the wormhole mouths they pass through.[3] The entrance mouth absorbing the spacecraft gains (equivalent) mass while the exit mouth emitting it loses (equivalent) mass.[†] (This mass coupling and conservation law takes into account the possibility that spaceships traversing the wormhole may lose or gain some kinetic energy in the process, and it is assumed that the two mouths are sufficiently far apart that their mutual gravitational interaction is negligible.) This unusual result suggests, but does not prove, the possibility of a fundamental limit on the total mass that can traverse a wormhole. The coupled mass conservation law shows

[†] Similar coupling and conservation results hold for the case of electrically charged matter that traverses a (charged or uncharged) wormhole.

that for a sufficiently large net transfer of mass the final (equivalent) mass of the exit mouth becomes negative. This is actually a beneficial result because ANEC violations are required just to hold the wormhole throat open in the first place. If it appears that a runaway reaction might occur, then it would be prudent for wormhole engineers to simply "turn off" the wormhole for a brief moment and then "turn it back on" (i.e., "reset" the wormhole) to restart space transportation operations. It is on the basis of the foregoing discussion that we can say with confidence that traversable wormholes are the most viable form of FTL transport.

On physical grounds Eq. (9) appears to be the correct negative energy/energy condition violation quantifier. However, further work is needed to establish whether Eq. (9) is the correct quantifier to use overall and whether all (averaged) energy condition theorems can be extended to include it.

On another note, A. Borde, L. Ford and T. Roman[74] have recast the QI conjecture into a new program which seeks to study the allowed spatial distributions of negative energy density in quantum field theory. Their study models free (massless) scalar fields in flat 2-dimensional Minkowski spacetime. Several explicit examples of spacetime averaged QI were studied to allow or rule out some particular model (spatial) distributions of negative energy. Their analysis showed that some geometric configurations of negative energy can either be ruled out or else constrained by the QI restrictions placed upon the allowable spatial distributions of negative energy. And there were found to be allowable negative energy distributions in which observers would never encounter the accompanying positive energy distribution. The extent to which these results for a flat 2-dimensional spacetime can be generalized to 4-dimensional curved spacetime remains unsolved (i.e., it is not yet clear if this can be solved). Therefore, we can still safely ignore the QI.

C. Observing Negative Energy in the Lab

Negative energy should be observable in lab experiments. The presence of naturally occurring negative energy regions in space is predicted to produce a unique signature corresponding to lensing, chromaticity and intensity effects in micro- and macro-lensing events on galactic and extragalactic/cosmological scales.[75-80] It has been shown that these effects provide a specific signature that allows for discrimination between ordinary (positive mass-energy) and negative energy lenses via the spectral analysis of astronomical lensing events. Theoretical modeling of negative energy lensing effects has led to intense astronomical searches for naturally occurring traversable wormholes in the universe. Computer model simulations and comparison of their results with recent satellite observations of gamma ray bursts (GRBs) has shown that putative negative energy (i.e., traversable wormhole) lensing events very closely resemble the main features of some GRBs. Other research has found that current observational data suggest that large amounts of naturally occurring "exotic matter" must have existed sometime between the epoch of galaxy formation and the present in order to (properly) quantitatively account for the "age-of-the-oldest-stars-in-the-galactic halo" problem and the cosmological evolution parameters.[71]

When background light rays strike a negative energy lensing region, they are swept out of the central region thus creating an umbra region of zero intensity. At the edges of the umbra the rays accumulate and create a rainbow-like caustic with enhanced light intensity. The lensing of a negative energy region is not analogous to a diverging lens because in certain circumstances it can produce more light enhancement than does the lensing of an equivalent positive mass-energy region. Real background sources in lensing events can have non-uniform brightness distributions on their surfaces and a dependency of their emission with the observing frequency. These complications can result in chromaticity effects, i.e., in spectral changes induced by differential lensing during the event. The quantification of such effects is quite lengthy, somewhat model dependent, and with recent application only to astronomical lensing events. Suffice it to say that future work is necessary to scale down the predicted lensing parameters and characterize their effects for lab experiments in which the negative energy will not be of astronomical magnitude. Present ultrahigh-speed optics and optical cavities, lasers, photonic crystal (and switching) technology, sensitive nano-sensor technology, and other techniques are very likely capable of detecting the very small magnitude lensing effects expected in lab experiments.

A non-optical scheme for detecting negative energy in experiments was recently reported by P. C. W. Davies and A. C. Ottewill[81] who studied the response of switched particle detectors to static negative energy densities and negative energy fluxes. Their model is based on a free (massless) scalar field in flat 4-dimensional Minkowski spacetime and utilized a simple generalization of the standard monopole detector, which is switched on and off to concentrate the measurements on periods of isolated negative energy density (or negative energy flux). The detector model includes an explicit switching factor whereby five different switching functions (based on data windowing theory) are defined and evaluated. In order to isolate the effects of negative energy a comparison is made for the response of a detector switched on and off during a period of negative energy density (or negative energy flux) and that switched on and off in the vacuum. The results shed light on the response of matter (detectors) to pulses of negative energy of finite duration, and they showed that negative energy should have the effect of enhancing deexcitation (i.e., induce cooling) of the detector. This is the opposite of our experience with detectors which undergo excitation when encountering "normal" matter or energy, and isolated detectors placed in a vacuum naturally cool due to the usual thermodynamic reasons. But Davies and Ottewill point out that the enhanced cooling effect they discovered cannot be used to draw a thermodynamic conclusion because their modeling was restricted to first order in perturbation theory. It is not possible at first order to determine whether the enhanced cooling effects are due to the small violation of energy conservation expected in any process in which a general quantum state collapses to an energy eigenstate, or whether they predict a systematic reduction in the energy of the detector which has serious thermodynamic implications. However, Davies and Ottewill point out that their results are model dependent and they found for their standard monopole detector model that there is not always a simple relationship between the strength of the negative energy density/flux and the behavior of the detector. Future work will be necessary to clear up these issues.

D. Time Machines and FTL Causality Issues

Simply put, any closed timelike curve (CTC), not necessarily a geodesic, in spacetime defines a time machine. Traversable wormholes imply time machines, and this discovery spawned an entirely new cottage industry of research on time machines.[2,3] Given a traversable wormhole, it appears to be very easy to build a time machine, although it will require a Herculean effort to induce a time shift between the two wormhole mouths (via Special Relativistic or General Relativistic time dilation techniques). Comprehensive theoretical studies show that the creation of a time machine might be the generic fate of a traversable wormhole.[3] One should not become alarmed by this revelation because it is a well known and widely accepted fact that classical (and semi-classical) general relativity is infested with time machines whereby there are numerous spacetime geometry solutions that exhibit time travel and/or have the properties of time machines.[3] It should be pointed out that various alternative theories of gravity, and quantum gravity models, are also infested with time machines, traversable wormholes, and warp drives. Because we are only interested in exploiting (static or dynamic) traversable wormholes to facilitate FTL travel between planets or stars we will not consider their time machine properties any further. It might be worth revisiting this topic only when a traversable wormhole can be successfully created in the lab (or in space).

Not surprisingly, warp drives also engender the appearance of CTCs.[31,82] Technical fixes have been proposed to mitigate against the formation of CTCs in and around the warp bubble by performing some minor modification(s) to the warp field geometry. However, since we previously showed that warp drives may not be technologically practical to implement, we will not consider this issue any further.

As a final note, there has been ongoing controversy within the physics community over the effects of FTL motion on causality. As this topic is beyond the scope of the present study, I will make only three points in this regard: 1) there are no grounds for microcausality violations in accordance with Ref. 83; 2) a new definition of causality is in order for FTL phenomena; and 3) investigators have found that CTCs/time machines do not affect Gauss's Theorem, and thus do not affect the derivation of global conservation laws from differential ones.[84] The standard conservation laws remain globally valid while retaining a natural quasi-local interpretation for spacetimes possessing time machines (for example, asymptotically flat wormhole spacetimes). K. Thorne[85] states that it may turn out that causality is violated at the macroscopic scale. Even if causality is obeyed macroscopically, then quantum gravity might offer finite probability amplitudes for microscopic spacetime histories possessing time machines. L. Li and J. Gott[86] found a self-consistent vacuum for quantum fields in Misner space (a simple flat space with CTCs) for which the renormalized stress-energy tensor is regular (in fact zero) everywhere. This implies that CTCs could exist at least at the level of semi-classical quantum gravity theory. Therefore, FTL causality paradoxes are just a reflection of our ignorance or inadequate comprehension of the physics of chronology and causality.

V. Conclusion

FTL spacetimes hold the promise of allowing mankind to reach the far planets and stars in our galaxy within a human lifetime in order to fulfill "the compelling urge of man to explore and to discover" which is guided by "the thrust of curiosity that leads men to try to go where no one has gone before."[87] In this assessment of FTL spacetimes we delineated a substantial number of crucial technical details along with their accompanying "make or break" issues having either tentative or firm results. The tentative results suggest a path for future research that is needed to put certain crucial technical issues on a firmer footing. We identified the two primary forms of FTL spacetimes found in GTR which can be created in principle: traversable wormholes and warp drives. These specialized spacetimes require the introduction of negative energy densities or fluxes in order to implement their geometries and FTL effects. Our assessment concludes that we already make small amounts of negative energy in the lab, but we do not yet know if we can access larger amounts for extended periods of time over extended spatial distributions for the purpose of engineering a particular FTL spacetime. We found that there are proposals for observing negative energy in outer space and in the lab, but further work is needed to downscale astronomical techniques for use at the lab scale, and we need to firm up our understanding of how lab detectors will respond to negative energy in situ. A detailed energy analysis showed that warp drives are not technologically practical to implement due to their requirement for extremely large amounts of negative energy (in order to achieve absurdly low "warp speeds"), while traversable wormholes appear to be the most practical to implement because of their very minimal negative energy requirement. Space limitations prevented us from giving a detailed discussion and analysis of FTL spacetimes within the framework of alternative theories of gravity or quantum gravity theories. Such theories remain speculative at present due to a lack of experimental validation, but we encourage the interested investigator to pursue this line of research to find any comparisons or contrasts with the results obtained from Einstein's GTR. We hope that this work will serve as a useful guide for the interested investigator who wishes to pursue this topic on their own.

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