Analytic Representations for the Dirac and Square-root Operators Of Relativistic Quantum Theory

Abstract

In this talk, I give a survey of recent work done on the Dirac and square-root equations. I first show how we can construct an analytical separation (diagonalization) of the full (minimal coupling) Dirac equation into particle and antiparticle components. The diagonalization is analytic in that it is achieved without transforming the wave function, as is done by the Foldy-Wouthuysen method, and reveals the nonlocal time behavior of the particle-antiparticle relationship. The natural interpretation of the zitterbewegung and the result that a velocity measurement (of a Dirac particle) at any instant in time is $\pm c$, seems to be that the Dirac equation makes a spatially extended particle appear as a point in the present by forcing it to oscillate between the past and future at speed c.

I then discuss the recent construction of an analytic representation for the square-root operator of relativistic quantum theory. We find that the square-root operator is represented by a non-local composite of three singularities. This effect is confined within a Compton wavelength such that, at the point of singularity, the three singularities cancel each other providing a finite result. Furthermore, the operator looks (almost) like the identity outside a Compton wavelength, but has a residual instantaneous connection with the whole universe at each point in time (spatial non-locality). To our knowledge, this is the first example of a physically relevant operator with these properties. In the standard interpretation, the particle component has two negative parts and one (hard core) positive part, while the antiparticle component has two positive parts and one (hard core) negative part.

Finally, I show that if we relinquish Minkowski's postulate, that the vector and scalar potentials be treated as components of a four-vector, we can obtain an alternate relationship between the Dirac equation (with minimal coupling) and the square-root equation, which is much closer than the one obtained via the Foldy-Wouthuysen method, in that there is no transformation of the wave function. This is accomplished by considering the scalar potential to be a part of the mass. This approach leads to a new Klein-Gordon (type) equation and a new square-root equation, both of which can have the same eigenfunctions as the Dirac equation.