Thermal Analysis of ETI Dewar Design

There are two main modes of heat transfer in the dewar design. The first is conduction through the components that connect the cold zone to the outside. The chief components of that are the inner cylindrical walls of the dewar and the drive shaft tube. The second mode is a heat leak through the dewar walls themselves. This can either occur via radiation if the dewar is simply evacuated or through "effective" conduction if the dewar walls contain MLI. Both cases will be analyzed.

Dewar specifics:

$l_i := 167.1 \cdot mm$ — length of inner wall, along which conduction occurs

$l_o := 185.1 \cdot mm$ — length of outer wall

$r_i := \frac{160}{2} \cdot mm$ — radius of inner bottom plate

$r_o := \frac{196}{2} \cdot mm$ — radius of outer bottom plate

$t_b := 3 \cdot mm$ — thickness of bottom plates

$t_i := 0.5 \cdot mm$ — thickness of inner wall

$area_i := 2 \cdot \pi \cdot r_i \cdot l_i + \pi \cdot r_i^2$ — surface area of the inner cylinder (walls and bottom)

$area_o := 2 \cdot \pi \cdot r_o \cdot l_o + \pi r_o^2$ — surface area of the inner cylinder (walls and bottom)

Conduction up the walls of the dewar and the drive tube

$K_{int} := 3.06 \cdot 10^3 \cdot \frac{W}{m}$ — integral of thermal conductivity of SS304 from 4K to 300K (*Experimental Techniques for Low-Temperature Measurement*, Ekin, Appendix A2.1)

Walls:

$cross_i := \pi \cdot \left[r_i^2 - \left(r_i - t_i \right)^2 \right]$ — cross sectional area of inner cylinder

$$q_{ci} := \frac{cross_i \cdot K_{int}}{\frac{l_i}{2}}$$

heat loss through conduction assuming that the dewar is at 4K halfway up its length

$q_{ci} = 9.2\,W$

Drive tube:

$l_d := 158.75 \cdot mm$ — length of tube

$r_d := \frac{50.8}{2} \cdot mm$ — outer radius of tube

$t_d := 0.762 \cdot mm$ thickness of tube

$cross_d := \pi \cdot \left[r_d^{\,2} - \left(r_d - t_d\right)^2 \right]$ cross sectional area of drive tube

$$q_{cd} := \frac{cross_d \cdot K_{int}}{\frac{l_d}{2}}$$

heat loss through conduction assuming that the tube is at 4K halfway up its length

$q_{cd} = 4.6\,W$

Heat loss through dewar walls

Without MLI: Radiation in a two surface enclosure

$\varepsilon := 0.03$ emissivity of polished metal (eg bright chrome plating)

$T_i := 4 \cdot K$ assume the entire inner wall is at 4K

$T_o := 295 \cdot K$ assume the entire outer wall is at RT

$$q_r := \frac{\sigma \cdot \left(T_o^{\,4} - T_i^{\,4}\right)}{\frac{1-\varepsilon}{\varepsilon \cdot area_i} + \frac{1}{area_i} + \frac{1-\varepsilon}{\varepsilon \cdot area_o}}$$

$q_r = 0.789\,W$

With MLI: Heat loss is still radiative, but common practice is to employ an effective conductivity for MLI

$k_{mli} := 100 \cdot 10^{-6} \cdot \frac{W}{m \cdot K}$ typical thermal conductivity of MLI

$t_{mli} := 1 \cdot cm$ thickness of MLI

$$q_{mli} := \frac{k_{mli} \cdot area_o \cdot \left(T_o - T_i\right)}{t_{mli}}$$

$q_{mli} = 0.419\,W$

The addition of MLI does not at first appear to be a critical factor and we will continue with the assumption that we do not need to use it.

Total heat leak:

$$q_{total} := q_r + q_{ci} + q_{cd}$$

$$q_{total} = 14.583\,W$$

In *Experimental Techniques in Low Temperature Physics*, White and Meeson, page 96, the authors state that an order of magnitude reduction in the heat conduction along thin metal walls can be achieved if the escaping gases are used to cool the metal. Using this technique, we can reduce the heat leak to:

$$q_{total\_2} := q_r + \frac{q_{ci} + q_{cd}}{10}$$

$$q_{total\_2} = 2.168\,W$$

Liquid helium requirements:

$$h := 83\cdot\frac{joule}{4\cdot gm}$$ heat of vaporization of helium

$$\rho_{LHe} := .125\cdot\frac{gm}{mL}$$ density of liquid He at the boiling point

$$H := h\cdot\rho_{LHe}$$

$$H = 2.594\times 10^{3}\,\frac{joule}{liter}$$ heat of vaporization in joules/liter

LHe usage if we don't use cooling from escaping He gas:

$$He := \frac{q_{total}}{H}$$

$$He = 20.24\,\frac{liter}{hr}$$

LHe usage when using escaping He gas for cooling:

$$He_2 := \frac{q_{total\_2}}{H}$$

$$He_2 = 3.009\,\frac{liter}{hr}$$

Reference:C:\Documents and Settings\marissa\My Documents\constants.mcd